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# Sfermion masses in the supersymmetric economical 3-3-1 model

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Abstract: Sfermion masses and eigenstates in the supersymmetric economical 3-3-1 model are studied. By lepton number conservation, the exotic squarks and superpartners of ordinary quarks are decoupled. Due to the fact that in the 3-3-1 models, one generation of quarks behaves differently from other two, by $R$-parity conservation, the mass mixing matrix of the squarks in this model are smaller than that in the Minimal Supersymmetric Standard Model (MSSM). Assuming substantial mixing in pairs of highest flavours, we are able to get mass spectrum and eigenstates of all the sfermions. In the effective approximation, the slepton mass splittings in the first two generations, are consistent with those in the MSSM, namely: $m^2_{\tilde{l}_L} - m^2_{\tilde{\nu}_{lL}} = m^2_W \cos 2\gamma$ $(l = e,\ \mu)$. In addition, within the above effective limit, there exists degeneracy among sneutrinos in each multiplet: $m^2_{\tilde{\nu}_{lL}} = m^2_{\tilde{\nu}_{lR}}$. In contradiction to the MSSM, the squark mass splittings are different for each generation and not to be $m^2_W \cos 2\gamma$.

Keywords: Supersymmetric partners of known particles, Models beyond the standard model.

## Contents



## 1. Introduction

The Standard Model (SM) of high energy physics provides a remarkable successful description of presently known phenomena. In spite of these successes, it fails to explain several fundamental issues like generation number puzzle, neutrino masses and oscillations, the origin of charge quantization, CP violation, etc.

One of the simplest solutions to these problems is to enhance the SM symmetry $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ to $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$ (called 3-3-1 for short) [1, 2, 3] gauge group. One of the main motivations to study this kind of models is an explanation in part of the generation number puzzle. In the 3-3-1 models, each generation is not anomaly free; and the model becomes anomaly free if one of quark families behaves differently from

other two. Consequently, the number of generations is multiple of the color number. Combining with the QCD asymptotic freedom, the generation number has to be three. For the neutrino masses and oscillations, the electric charge quantization and CP violation issues in the 3-3-1 models, the interested readers can find in Refs. [4], [5] and [6], respectively.

In one of the 3-3-1 models, the right-handed neutrinos are in bottom of the lepton triplets [3] and three Higgs triplets are required. It is worth noting that, there are two Higgs triplets with *neutral components in the top and bottom*. In the earlier version, these triplets can have vacuum expectation value (VEV) either on the top or in the bottom, but not in both. Assuming that all neutral components in the triplet can have VEVs, we are able to reduce number of triplets in the model to be two [7, 8]. Such a scalar sector is minimal, therefore it has been called the economical 3-3-1 model [9]. In a series of papers, we have developed and proved that this non-supersymmetric version is consistent, realistic and very rich in physics [8, 9, 10, 11].

In the other hands, due to the "no-go" theorem of Coleman-Mandula [12], the internal $G$ and external $P$ spacetime symmetries can only be *trivially* unified. In addition, the mere fact that the ratio $M_P/M_W$ is so huge is already a powerful clue to the character of physics beyond the SM, because of the infamous hierarchy problem. In the framework of new symmetry called a supersymmetry [13, 14], the above mentioned problems can be solved. One of the intriguing features of supersymmetric theories is that the Higgs spectrum (unfortunately, the only part of the SM is still not discovered) is quite constrained.

It is known that the economical (non-supersymmetric) 3-3-1 model does not furnish any candidate for self-interaction dark matter [15] with the condition given by Spergel and Steinhardt [16]. With a larger content of the scalar sector, the supersymmetric version is expected to have a candidate for the self-interaction dark matter. The supersymmetric version of the 3-3-1 model with right-handed neutrinos [3] has already been constructed in Refs. [17]. An supersymmetric version of the economical 3-3-1 model has been constructed in Ref. [18]. Some interesting features such as Higgs bosons with masses equal to that of the gauge bosons – the $W$ and the bileptons $X$ and $Y$, have been pointed out in Ref. [19].

In a supersymmetric extension of the (beyond) SM, each of the known fundamental particles must be in either a chiral or gauge supermultiplet and have a superpartner with spin differing by 1/2 unit. All of the matter fermions (the known quarks and leptons) have spin-0 partners called sfermions. Hence in supersymmetric models, besides scalar Higgs bosons, there are scalar sfermions. In Ref. [18], the Higgs sector was a subject of our interest; in this paper, we will focus an attention to the sfermions – sleptons and squarks.

This article is organized as follows. In Sec. 2 we present a fermion and scalar content in the supersymmetric economical 3-3-1 model. The necessary parts of Lagrangian is also given. The $F$ and $D$ terms of scalar potential for sfermions are calculated in Sec. 3. Masses and eigenstates for sleptons and squarks are given in Sec. 4 and 5, respectively. Section 6 is devoted for the case of $R$-parity conservation and sfermion mass splittings. Finally, we summarize our results and make conclusions in the last section - Sec. 7.

## 2. A review of the model

In this section we first recapitulate the basic elements of the supersymmetric economical 3-3-1 model [18]. $R - parity$ and some constraints on the couplings are also presented.

### 2.1 Particle content

The superfield content in this paper is defined in a standard way as follows

$$\widehat{F} = (\widetilde{F}, F), \qquad \widehat{S} = (S, \widetilde{S}), \qquad \widehat{V} = (\lambda, V), \tag{2.1}$$

where the components $F$, $S$ and $V$ stand for the fermion, scalar and vector fields while their superpartners are denoted as $\widetilde{F}$, $\widetilde{S}$ and $\lambda$, respectively [13, 17].

The superfield content in the considering model with an anomaly-free fermionic content transforms under the 3-3-1 gauge group as

$$\widehat{L}_{aL} = \left(\widehat{\nu}_a, \widehat{l}_a, \widehat{\nu}^c_a\right)^T_L \sim (1, 3, -1/3), \qquad \widehat{l}^c_{aL} \sim (1, 1, 1), \tag{2.2}$$

$$\widehat{Q}_{1L} = \left(\widehat{u}_1,\ \widehat{d}_1,\ \widehat{u}'\right)^T_L \sim (3, 3, 1/3), \tag{2.3}$$

$$\widehat{u}^c_{1L},\ \widehat{u}'^c_L \sim (3^*, 1, -2/3), \qquad \widehat{d}^c_{1L} \sim (3^*, 1, 1/3), \tag{2.4}$$

$$\widehat{Q}_{\alpha L} = \left(\widehat{d}_\alpha, -\widehat{u}_\alpha, \widehat{d}'_\alpha\right)^T_L \sim (3, 3^*, 0), \qquad \alpha = 2, 3, \tag{2.5}$$

$$\widehat{u}^c_{\alpha L} \sim (3^*, 1, -2/3)\,, \qquad \widehat{d}^c_{\alpha L},\ \widehat{d}'^c_{\alpha L} \sim (3^*, 1, 1/3)\,, \tag{2.6}$$

where the values in the parentheses denote quantum numbers based on ($\mathrm{SU}(3)_C$, $\mathrm{SU}(3)_L$, $\mathrm{U}(1)_X$) symmetry. $\widehat{\nu}^c_L = (\widehat{\nu}_R)^c$ and $a = 1, 2, 3$ is a generation index. The primes superscript on usual quark types ($u'$ with the electric charge $q_{u'} = 2/3$ and $d'$ with $q_{d'} = -1/3$) indicate that those quarks are exotic ones.

The two superfields $\widehat{\chi}$ and $\widehat{\rho}$ are at least introduced to span the scalar sector of the economical 3-3-1 model [9]:

$$\begin{aligned} \widehat{\chi} &= \left(\widehat{\chi}^0_1, \widehat{\chi}^-, \widehat{\chi}^0_2\right)^T \sim (1, 3, -1/3), && (2.7)\\ \widehat{\rho} &= \left(\widehat{\rho}^+_1, \widehat{\rho}^0, \widehat{\rho}^+_2\right)^T \sim (1, 3, 2/3). && (2.8) \end{aligned}$$

To cancel the chiral anomalies of Higgsino sector, the two extra superfields $\widehat{\chi}'$ and $\widehat{\rho}'$ must be added as follows

$$\begin{aligned} \widehat{\chi}' &= \left(\widehat{\chi}'^0_1, \widehat{\chi}'^+, \widehat{\chi}'^0_2\right)^T \sim (1, 3^*, 1/3), && (2.9)\\ \widehat{\rho}' &= \left(\widehat{\rho}'^-_1, \widehat{\rho}'^0, \widehat{\rho}'^-_2\right)^T \sim (1, 3^*, -2/3). && (2.10) \end{aligned}$$

In this model, the $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$ gauge group is broken via two steps:

$$\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X \stackrel{w,w'}{\longrightarrow} \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \stackrel{v,v',u,u'}{\longrightarrow} \mathrm{U}(1)_Q, \tag{2.11}$$

where the VEVs are defined by

$$\sqrt{2}\langle\chi\rangle^T = (u,0,w)\,, \qquad \sqrt{2}\langle\chi'\rangle^T = \big(u',0,w'\big)\,, \tag{2.12}$$

$$\sqrt{2}\langle\rho\rangle^T = (0,v,0)\,, \qquad \sqrt{2}\langle\rho'\rangle^T = \big(0,v',0\big)\,. \tag{2.13}$$

The VEVs $w$ and $w'$ are responsible for the first step of the symmetry breaking while $u,\ u'$ and $v,\ v'$ are for the second one. Therefore, they have to satisfy the constraints:

$$u,\ u',\ v,\ v' \ll w,\ w'. \tag{2.14}$$

The vector superfields $\widehat{V}_c$, $\widehat{V}$ and $\widehat{V}'$ containing the usual gauge bosons are, respectively, associated with the $\mathrm{SU}(3)_C$, $\mathrm{SU}(3)_L$ and $\mathrm{U}(1)_X$ group factors. The colour and flavour vector superfields have expansions in the Gell-Mann matrix bases $T^a = \lambda^a/2$ $(a = 1,2,...,8)$ as follows

$$\widehat{V}_c = \frac{1}{2}\lambda^a\widehat{V}_{ca}, \quad \widehat{\overline{V}}_c = -\frac{1}{2}\lambda^{a*}\widehat{V}_{ca}; \quad \widehat{V} = \frac{1}{2}\lambda^a\widehat{V}_a, \quad \widehat{\overline{V}} = -\frac{1}{2}\lambda^{a*}\widehat{V}_a, \tag{2.15}$$

where an overbar $^-$ indicates complex conjugation. For the vector superfield associated with $\mathrm{U}(1)_X$, we normalize as follows

$$X\hat{V}' = (XT^9)\hat{B}, \qquad T^9 \equiv \frac{1}{\sqrt{6}}\mathrm{diag}(1,1,1). \tag{2.16}$$

The gluons are denoted by $g^a$ and their respective gluino partners by $\lambda^a_c$, with $a = 1,\ldots,8$. In the electroweak sector, $V^a$ and $B$ stand for the $\mathrm{SU}(3)_L$ and $\mathrm{U}(1)_X$ gauge bosons with their gaugino partners $\lambda^a_V$ and $\lambda_B$, respectively.

With the superfields as given, the full Lagrangian is defined by $\mathcal{L}_{susy} + \mathcal{L}_{soft}$, where the first term is supersymmetric part, whereas the last term breaks explicitly the supersymmetry [18]. The interested reader can find more details on this Lagrangian in the above mentioned article. In the following, only terms relevant to our calculations are displayed.

From the supersymmetric Lagrangian [18], we can obtain the following superpotential

$$W = \frac{W_2}{2} + \frac{W_3}{3}, \tag{2.17}$$

where

$$W_2 = \mu_{0a}\hat{L}_{aL}\hat{\chi}' + \mu_\chi\hat{\chi}\hat{\chi}' + \mu_\rho\hat{\rho}\hat{\rho}', \tag{2.18}$$

and

$$\begin{aligned} W_3 = {} & \gamma_{ab}\hat{L}_{aL}\hat{\rho}'\hat{l}^c_{bL} + \lambda_a\epsilon\hat{L}_{aL}\hat{\chi}\hat{\rho} + \lambda'_{ab}\epsilon\hat{L}_{aL}\hat{L}_{bL}\hat{\rho} \\ & + \kappa_i\hat{Q}_{1L}\hat{\chi}'\hat{u}^c_{iL} + \kappa'\hat{Q}_{1L}\hat{\chi}'\hat{u}'^c_L + \vartheta_i\hat{Q}_{1L}\hat{\rho}'\hat{d}^c_{iL} \\ & + \vartheta'_\alpha\hat{Q}_{1L}\hat{\rho}'\hat{d}'^c_{\alpha L} + \pi_{\alpha i}\hat{Q}_{\alpha L}\hat{\rho}\hat{u}^c_{iL} + \pi'_\alpha\hat{Q}_{\alpha L}\hat{\rho}\hat{u}'^c_L \\ & + \Pi_{\alpha i}\hat{Q}_{\alpha L}\hat{\chi}\hat{d}^c_{iL} + \Pi'_{\alpha\beta}\hat{Q}_{\alpha L}\hat{\chi}\hat{d}'^c_{\beta L} + \epsilon f_{\alpha\beta\gamma}\hat{Q}_{\alpha L}\hat{Q}_{\beta L}\hat{Q}_{\gamma L} \\ & + \xi_{1i\beta j}\hat{d}^c_{iL}\hat{d}'^c_{\beta L}\hat{u}^c_{jL} + \xi_{2i\beta}\hat{d}^c_{iL}\hat{d}'^c_{\beta L}\hat{u}'^c_L + \xi_{3ijk}\hat{d}^c_{iL}\hat{d}^c_{jL}\hat{u}^c_{kL} \\ & + \xi_{4ij}\hat{d}^c_{iL}\hat{d}^c_{jL}\hat{u}'^c_L + \xi_{5\alpha\beta i}\hat{d}'^c_{\alpha L}\hat{d}'^c_{\beta L}\hat{u}^c_{iL} + \xi_{6\alpha\beta}\hat{d}'^c_{\alpha L}\hat{d}'^c_{\beta L}\hat{u}'^c_L \\ & + \xi_{a\alpha j}\hat{L}_{aL}\hat{Q}_{\alpha L}\hat{d}^c_{jL} + \xi'_{a\alpha\beta}\hat{L}_{aL}\hat{Q}_{\alpha L}\hat{d}'^c_{\beta L}. \end{aligned} \tag{2.19}$$

The coefficients $\mu_{0a}, \mu_\rho$ and $\mu_\chi$ have mass dimension, while all coefficients in $W_3$ are dimensionless and $\lambda'_{ab} = -\lambda'_{ba}$.

It is worth noting that the first term of (2.19) is the Yukawa coupling giving charged leptons mass, while the third one is responsible for neutrino mass. At the tree level, their couplings satisfy the following estimation [11, 20]:

$$\gamma_{ab} \gg \lambda'_{ab}. \tag{2.20}$$

In the SM, neutrinos are rigid massless, hence $\lambda'_{ab}$ has to be vanish. In other words, we can put $\lambda'_{ab} = 0$ in the SM limit.

From the soft supersymmetry-breaking terms [18], the Lagangian relevant to the sfermions is obtained by

$$\begin{aligned} -\mathcal{L}_{SMT} = & M^2_{ab}\widetilde{L}^\dagger_{aL}\widetilde{L}_{bL} + m^2_{ab}\widetilde{l}^{c*}_{aL}\widetilde{l}^c_{bL} + m^2_{Q1L}\widetilde{Q}^\dagger_{1L}\widetilde{Q}_{1L} + m^2_{Q\alpha\beta L}\widetilde{Q}^\dagger_{\alpha L}\widetilde{Q}_{\beta L} \\ & + m^2_{u_{ij}}\widetilde{u}^{c*}_{iL}\widetilde{u}^c_{jL} + m^2_{d_{ij}}\widetilde{d}^{c*}_{iL}\widetilde{d}^c_{jL} + m^2_{u'}\widetilde{u'}^{c*}_L\widetilde{u'}^c_L + m^2_{d'\alpha\beta}\widetilde{d'}^{c*}_{\alpha L}\widetilde{d'}^c_{\beta L} \\ & + \Big\{ M'^2_a\chi^\dagger\widetilde{L}_{aL} + \eta_{ab}\widetilde{L}_{aL}\rho'\widetilde{l}^c_{Lb} + \upsilon_a\epsilon\widetilde{L}_{aL}\chi\rho + \varepsilon_{ab}\epsilon\widetilde{L}_{aL}\widetilde{L}_{bL}\rho + p_i\widetilde{Q}_{1L}\chi'\widetilde{u}^c_{iL} \\ & + p\widetilde{Q}_{1L}\chi'\widetilde{u'}^c_L + p_{\alpha i}\widetilde{Q}_{\alpha L}\rho\widetilde{u}^c_{iL} + r_\alpha\widetilde{Q}_{\alpha L}\rho\widetilde{u'}^c_L + h_i\widetilde{Q}_{1L}\rho'\widetilde{d}^c_{iL} \\ & + h'_i\widetilde{Q}_{1L}\rho'\widetilde{d'}^c_{iL} + h_{\alpha i}\widetilde{Q}_{\alpha L}\chi\widetilde{d}^c_{iL} + h'_{\alpha\beta}\widetilde{Q}_{\alpha L}\chi\widetilde{d'}^c_{\beta L} \\ & + p_{5\alpha\beta\gamma}\widetilde{Q}_{\alpha L}\widetilde{Q}_{\beta L}\widetilde{Q}_{\gamma L} + \kappa_{i\beta j}\widetilde{d}^c_{iL}\widetilde{d'}^c_{\beta L}\widetilde{u}^c_{jL} + \vartheta_{i\beta}\widetilde{d}^c_{iL}\widetilde{d'}^c_{\beta L}\widetilde{u'}^c_L \\ & + \pi_{ijk}\widetilde{d}^c_{iL}\widetilde{d}^c_{jL}\widetilde{u}^c_{kL} + \kappa_{4ik}\widetilde{d}^c_{iL}\widetilde{d}^c_{jL}\widetilde{u'}^c_L + \kappa_{5\alpha\beta i}\widetilde{d'}^c_{\alpha L}\widetilde{d'}^c_{\beta L}\widetilde{u}^c_{iL} \\ & + \kappa_{6\alpha\beta}\widetilde{d'}^c_{\alpha L}\widetilde{d'}^c_{\beta L}\widetilde{u'}^c_L + \omega_{a\alpha j}\widetilde{L}_{aL}\widetilde{Q}_{\alpha L}\widetilde{d}^c_{jL} + \omega'_{a\alpha\beta}\widetilde{L}_{aL}\widetilde{Q}_{\alpha L}\widetilde{d'}^c_{\beta L} + H.c. \Big\}, \end{aligned} \tag{2.21}$$

where $\varepsilon_{ab} = -\varepsilon_{ba}$. This Lagrangian is also responsible for sfermion masses.

### 2.2 $R$-parity

For the further analysis, it is convenience to introduce $R$-parity in the model. Following Ref. [20], $R$-parity can be expressed as follows

$$R - parity = (-1)^{2S}(-1)^{3(\mathcal{B}+\mathcal{L})} \tag{2.22}$$

where invariant charges $\mathcal{L}$ and $\mathcal{B}$ (for details, see Ref. [21]) are given by

| $Triplet$ | $L$ | $Q_1$ | $\chi$ | $\rho$ |
|---|---|---|---|---|
| $\mathcal{B}$ $charge$ | 0 | $\frac{1}{3}$ | 0 | 0 |
| $\mathcal{L}$ $charge$ | $\frac{1}{3}$ | $-\frac{2}{3}$ | $\frac{4}{3}$ | $-\frac{2}{3}$ |

(2.23)

| $Anti-Triplet$ | $Q_\alpha$ | $\chi'$ | $\rho'$ |
|---|---|---|---|
| $\mathcal{B}$ $charge$ | $\frac{1}{3}$ | 0 | 0 |
| $\mathcal{L}$ $charge$ | $\frac{2}{3}$ | $-\frac{4}{3}$ | $\frac{2}{3}$ |

(2.24)

| $Singlet$ | $l^c$ | $u^c$ | $d^c$ | $u'^c$ | $d'^c$ |
|---|---|---|---|---|---|
| $\mathcal{B}$ $charge$ | 0 | $-\frac{1}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ |
| $\mathcal{L}$ $charge$ | $-1$ | 0 | 0 | 2 | $-2$ |

(2.25)

Combining (2.22) and the above tables, it is easy to conclude that the fields $\chi$, $\chi'$, $\rho$, $\rho'$, $L$, $Q_\alpha$, $Q_3$, $l$, $u$, $u'$, $d$ and $d'$ have $R$-charge equal to one, while their superpartners have opposite $R$-charge, as in the Minimal Supersymmetric Standard Model (MSSM).

Under $R$-parity transformation, the Higgs and matter superfields change, respectively [20]:

$$\begin{aligned}\hat{H}_{1,2}(x,\theta,\bar{\theta}) &\xmapsto{\mathbf{R}_d} \hat{H}_{1,2}(x,-\theta,-\bar{\theta}),\\ \hat{\Phi}(x,\theta,\bar{\theta}) &\xmapsto{\mathbf{R}_d} -\hat{\Phi}(x,-\theta,-\bar{\theta}),\ \ \Phi = Q, u^c, d^c, L, l^c,\end{aligned} \tag{2.26}$$

Let us separate $W$ and $\mathcal{L}_{SMT}$ into the $R$-parity conserving ($R$) and violating ($\not{R}$) part. Thus

$$W = W_R + W_{\not{R}}, \tag{2.27}$$

where

$$\begin{aligned} W_R = &\frac{1}{2}\left(\mu_\chi\hat{\chi}\hat{\chi}' + \mu_\rho\hat{\rho}\hat{\rho}'\right)\\ &+\frac{1}{3}\Big(\gamma_{ab}\hat{L}_{aL}\hat{\rho}'\hat{l}^c_{bL} + \lambda'_{ab}\epsilon\hat{L}_{aL}\hat{L}_{bL}\hat{\rho}\\ &+\kappa'\hat{Q}_{1L}\hat{\chi}'\hat{u}'^c_L + \vartheta_i\hat{Q}_{1L}\hat{\rho}'\hat{d}^c_{iL} + \pi_{\alpha i}\hat{Q}_{\alpha L}\hat{\rho}\hat{u}^c_{iL} + \Pi_{\alpha i}\hat{Q}_{\alpha L}\hat{\chi}\hat{d}^c_{iL}\\ &+\kappa_i\hat{Q}_{1L}\hat{\chi}'\hat{u}^c_{iL} + \vartheta'_\alpha\hat{Q}_{1L}\hat{\rho}'\hat{d}'^c_{\alpha L} + \pi'_\alpha\hat{Q}_{\alpha L}\hat{\rho}\hat{u}'^c_L + \Pi'_{\alpha\beta}\hat{Q}_{\alpha L}\hat{\chi}\hat{d}'^c_{\beta L}\Big), \end{aligned} \tag{2.28}$$

and

$$\begin{aligned} W_{\not{R}} = &\frac{1}{2}\mu_{0a}\hat{L}_{aL}\hat{\chi}' + \frac{1}{3}\Big(\lambda_a\epsilon\hat{L}_{aL}\hat{\chi}\hat{\rho} + \epsilon f_{\alpha\beta\gamma}\hat{Q}_{\alpha L}\hat{Q}_{\beta L}\hat{Q}_{\gamma L}\\ &+\xi_{1i\beta j}\hat{d}^c_{iL}\hat{d}'^c_{\beta L}\hat{u}^c_{jL} + \xi_{2i\beta}\hat{d}^c_{iL}\hat{d}'^c_{\beta L}\hat{u}'^c_L + \xi_{3ijk}\hat{d}^c_{iL}\hat{d}^c_{jL}\hat{u}^c_{kL}\\ &+\xi_{4ij}\hat{d}^c_{iL}\hat{d}^c_{jL}\hat{u}'^c_L + \xi_{5\alpha\beta i}\hat{d}'^c_{\alpha L}\hat{d}'^c_{\beta L}\hat{u}^c_{iL} + \xi_{6\alpha\beta}\hat{d}'^c_{\alpha L}\hat{d}'^c_{\beta L}\hat{u}'^c_L\\ &+\xi_{a\alpha j}\hat{L}_{aL}\hat{Q}_{\alpha L}\hat{d}^c_{jL} + \xi'_{a\alpha\beta}\hat{L}_{aL}\hat{Q}_{\alpha L}\hat{d}'^c_{\beta L}\Big). \end{aligned} \tag{2.29}$$

By (2.26), the $\not{R}$ part contains odd number of *matter* superfields. For the soft terms, we have also

$$\mathcal{L}_{SMT} = \mathcal{L}^R_{SMT} + \mathcal{L}^{\not{R}}_{SMT}, \tag{2.30}$$

where

$$\begin{aligned} -\mathcal{L}^R_{SMT} = &M^2_{ab}\widetilde{L}^\dagger_{aL}\widetilde{L}_{bL} + m^2_{ab}\widetilde{l}^{c*}_{aL}\widetilde{l}^c_{bL} + m^2_{Q1L}\widetilde{Q}^\dagger_{1L}\widetilde{Q}_{1L} + m^2_{Q\alpha\beta L}\widetilde{Q}^\dagger_{\alpha L}\widetilde{Q}_{\beta L}\\ &+m^2_{u_{ij}}\widetilde{u}^{c*}_{iL}\widetilde{u}^c_{jL} + m^2_{d_{ij}}\widetilde{d}^{c*}_{iL}\widetilde{d}^c_{jL} + m^2_{u'}\widetilde{u'}^{c*}_L\widetilde{u'}^c_L + m^2_{d'\alpha\beta}\widetilde{d'}^{c*}_{\alpha L}\widetilde{d'}^c_{\beta L}\\ &+\Big\{\eta_{ab}\widetilde{L}_{aL}\rho'\widetilde{l}^c_{Lb} + \varepsilon_{ab}\epsilon\widetilde{L}_{aL}\widetilde{L}_{bL}\rho + p\widetilde{Q}_{1L}\chi'\widetilde{u'}^c_L\\ &+p_{\alpha i}\widetilde{Q}_{\alpha L}\rho\widetilde{u}^c_{iL} + h_i\widetilde{Q}_{1L}\rho'\widetilde{d}^c_{iL} + h_{\alpha i}\widetilde{Q}_{\alpha L}\chi\widetilde{d}^c_{iL}\\ &+p_i\widetilde{Q}_{1L}\chi'\widetilde{u}^c_{iL} + h'_\alpha\widetilde{Q}_{1L}\rho'\widetilde{d'}^c_{\alpha L} + h'_{\alpha\beta}\widetilde{Q}_{\alpha L}\chi\widetilde{d'}^c_{\beta L} + r_\alpha\widetilde{Q}_{\alpha L}\rho\widetilde{u'}^c_L + H.c.\Big\}, \end{aligned} \tag{2.31}$$

and

$$\begin{aligned} -\mathcal{L}^{\not{R}}_{SMT} = &M'^2_a\chi^\dagger\widetilde{L}_{aL} + \upsilon_a\epsilon\widetilde{L}_{aL}\chi\rho\\ &+p_{5\alpha\beta\gamma}\widetilde{Q}_{\alpha L}\widetilde{Q}_{\beta L}\widetilde{Q}_{\gamma L} + \kappa_{i\beta j}\widetilde{d}^c_{iL}\widetilde{d'}^c_{\beta L}\widetilde{u}^c_{jL} + \vartheta_{i\beta}\widetilde{d}^c_{iL}\widetilde{d'}^c_{\beta L}\widetilde{u'}^c_L\\ &+\pi_{ijk}\widetilde{d}^c_{iL}\widetilde{d}^c_{jL}\widetilde{u}^c_{kL} + \kappa_{4ik}\widetilde{d}^c_{iL}\widetilde{d}^c_{jL}\widetilde{u'}^c_L + \kappa_{5\alpha\beta i}\widetilde{d'}^c_{\alpha L}\widetilde{d'}^c_{\beta L}\widetilde{u}^c_{iL}\\ &+\kappa_{6\alpha\beta}\widetilde{d'}^c_{\alpha L}\widetilde{d'}^c_{\beta L}\widetilde{u'}^c_L + \omega_{a\alpha j}\widetilde{L}_{aL}\widetilde{Q}_{\alpha L}\widetilde{d}^c_{jL} + \omega'_{a\alpha\beta}\widetilde{L}_{aL}\widetilde{Q}_{\alpha L}\widetilde{d'}^c_{\beta L} + H.c. \end{aligned} \tag{2.32}$$

The $\not R$ soft terms consist of odd number of *supersymmetric partners* - sfermions.

Note that the last lines in (2.28) and (2.31) contain lepton-number violating terms (with $\Delta L = \pm 2$). Hence we have (see also [11])

$$\kappa_i, \vartheta'_\alpha, \pi'_\alpha, \Pi'_{\alpha\beta}, p_i, r_\alpha, h'_\alpha, h'_{\alpha\beta} \ll \kappa', \vartheta_i, \pi_{\alpha i}, \Pi_{\alpha i}, p, p_{\alpha i}, h_i, h_{\alpha i}. \tag{2.33}$$

## 3. Scalar potential for sfermions

The scalar potential of the model is a result of summation over $F$ and $D$ terms:

$$V = F^{\phi *}F_\phi + \frac{1}{2}\sum_a D^a D_a, \tag{3.1}$$

where [14]

$$F_\phi = \frac{\partial W}{\partial \phi}, \qquad W = W_2 + W_3, \tag{3.2}$$

and

$$D^a = -g\left(\sum_\phi \phi^* T^a \phi\right). \tag{3.3}$$

The field $\phi$ stands for all the scalars or sfermions in the model.

### 3.1 F-term contribution

From $W_2$ and $W_3$ we get

$$\begin{aligned}
F_{\chi'} &= \frac{1}{2}(\mu_{0a}\widetilde{L}_{aL} + \mu_\chi \chi) + \frac{1}{3}(\kappa_i \tilde{Q}_{1L}\tilde{u}^c_{iL} + \kappa' \tilde{Q}_{1L}\tilde{u}'^c_L), && (3.4)\\
F_{\chi^\sigma} &= \frac{1}{2}\mu_\chi \chi'_\sigma + \frac{1}{3}\left(\lambda_a \epsilon_{m\sigma n}\tilde{L}^m_{aL}\rho^n + \Pi_{\alpha i}\tilde{Q}_{\alpha L\sigma}\tilde{d}^c_{iL} + \Pi'_{\alpha\beta}\tilde{Q}_{\alpha L\sigma}\tilde{d}'^c_{\beta L}\right), && (3.5)\\
F_{\rho^\sigma} &= \frac{1}{2}\mu_\rho \rho'_\sigma + \frac{1}{3}\left(\lambda_a \epsilon_{mn\sigma}\tilde{L}^m_{aL}\chi^n + \lambda'_{ab}\epsilon_{mn\sigma}\tilde{L}^m_{aL}\tilde{L}^n_{bL}\right.\\
&\qquad \left. + \pi_{\alpha i}\tilde{Q}_{\alpha L\sigma}\tilde{u}^c_{iL} + \pi'_\alpha \tilde{Q}_{\alpha L\sigma}\tilde{u}'^c_L\right), && (3.6)\\
F_{\rho'} &= \frac{1}{2}\mu_\rho \rho + \frac{1}{3}\left(\gamma_{ab}\tilde{L}_{aL}\tilde{l}^c_{bL} + \vartheta_i \tilde{Q}_{1L}\tilde{d}^c_{iL} + \vartheta'_\alpha \tilde{Q}_{1L}\tilde{d}'^c_{\alpha L}\right), && (3.7)\\
F_{L^\sigma_{aL}} &= \frac{1}{2}\mu_{0a}\chi'_\sigma + \frac{1}{3}(\gamma_{ab}\rho'_\sigma \tilde{l}^c_{bL} + \lambda_a \epsilon_{\sigma mn}\chi^m \rho^n + 2\lambda'_{ab}\epsilon_{\sigma mn}\tilde{L}^m_{bL}\rho^n\\
&\qquad + \xi_{a\alpha j}\tilde{Q}_{\alpha L\sigma}\tilde{d}^c_{jL} + \xi'_{a\alpha\beta}\tilde{Q}_{\alpha L\sigma}\tilde{d}'^c_{\beta L}) && (3.8)\\
F_{l^c_{Lb}} &= \frac{1}{3}\gamma_{ab}\tilde{L}_{aL}\rho', && (3.9)\\
F_{Q_{1L}} &= \frac{1}{3}\left(\kappa_i \chi' \tilde{u}^c_{iL} + \kappa' \chi' \tilde{u}'^c_L + \vartheta_i \rho' \tilde{d}^c_{iL} + \vartheta'_\alpha \rho' \tilde{d}'^c_{\alpha L}\right), && (3.10)\\
F_{Q^\sigma_{\alpha L}} &= \frac{1}{3}\left(\pi_{\alpha i}\rho_\sigma \tilde{u}^c_{iL} + \pi'_\alpha \rho_\sigma \tilde{u}'^c_L + \Pi_{\alpha i}\chi_\sigma \tilde{d}^c_{iL} + \Pi'_{\alpha\beta}\chi_\sigma \tilde{d}'^c_{\beta L}\right.\\
&\qquad \left. + 3f_{\alpha\beta\gamma}\epsilon_{\sigma jk}\widetilde{Q}^j_{\beta L}\widetilde{Q}^k_{\gamma L} + \xi_{a\alpha i}\widetilde{L}_{aL\sigma}\tilde{d}^c_{iL} + \xi'_{a\alpha\beta}\widetilde{L}_{aL\sigma}\tilde{d}'^c_{\beta L}\right), && (3.11)\\
F_{u^c_{iL}} &= \frac{1}{3}\left(\kappa_i \chi' \widetilde{Q}_{1L} + \pi_{\alpha i}\rho \widetilde{Q}_{\alpha L} + \xi_{1j\beta i}\tilde{d}^c_{jL}\tilde{d}'^c_{\beta L} + \xi_{3kji}\tilde{d}^c_{kL}\tilde{d}^c_{jL} + \xi_{5\alpha\beta i}\tilde{d}'^c_{\alpha L}\tilde{d}'^c_{\beta L}\right), && (3.12)
\end{aligned}$$

$$F_{u'^c_L} = \frac{1}{3}\left(\kappa'\chi'\widetilde{Q}_{1L} + \pi'_\alpha\rho\widetilde{Q}_{\alpha L} + \xi_{2i\beta}\tilde{d}^c_{iL}\tilde{d'}^c_{\beta L} + \xi_{4ij}\tilde{d}^c_{iL}\tilde{d}^c_{jL} + \xi_{6\alpha\beta}\tilde{d'}^c_{\alpha L}\tilde{d'}^c_{\beta L}\right), \quad (3.13)$$

$$F_{d^c_{iL}} = \frac{1}{3}\left(\vartheta_i\rho'\widetilde{Q}_{1L} + \Pi_{\alpha i}\chi\widetilde{Q}_{\alpha L} + \xi_{1i\beta j}\tilde{d'}^c_{\beta L}\tilde{u}^c_{jL} + \xi_{2i\beta}\tilde{d'}^c_{\beta L}\tilde{u'}^c_L \right.$$
$$\left. + 2\xi_{3ijk}\tilde{d}^c_{jL}\tilde{u}^c_{kL} + 2\xi_{4ij}\tilde{d}^c_{jL}\tilde{u'}^c_L + \xi_{a\alpha i}\tilde{L}_{aL}\tilde{Q}_{\alpha L}\right), \quad (3.14)$$

$$F_{d'^c_{\alpha L}} = \frac{1}{3}\left(\vartheta'_\alpha\rho'\widetilde{Q}_{1L} + \Pi'_{\beta\alpha}\chi\widetilde{Q}_{\beta L} + \xi_{1i\alpha j}\tilde{d}^c_{iL}\tilde{u}^c_{jL} + \xi_{2i\alpha}\tilde{d}^c_{iL}\tilde{u'}^c_L \right.$$
$$\left. + 2\xi_{5\alpha\beta}\tilde{d'}^c_{\beta L}\tilde{u}^c_{iL} + 2\xi_{6\alpha\beta}\tilde{d'}^c_{\beta L}\tilde{u'}^c_L + \xi'_{a\beta\alpha}\tilde{L}_{aL}\tilde{Q}_{\beta L}\right). \quad (3.15)$$

With these $F$-terms, besides the second order mass terms in $V$, we also get trilinear and quartic couplings of the sfermions. Below only the mass terms and the linear (by fields) terms are our interest.

### 3.2 D-term contribution

By Eq. (3.3), we separate two subgroups, namely $SU(3)_L$ and $U(1)_X$.

1. *D-term contribution from* $SU(3)_L$:

   The interested contribution to sfermion masses has a form

$$D^a = -g\left[\sum_{sfermions}\tilde{f}^\dagger T^a\tilde{f} + \sum_{Higgs} H^\dagger T^a H\right]. \quad (3.16)$$

   Since $T_a = T_a^\dagger$, we have

$$(D^a)^* D_a = 2g^2\left[\left(\sum_{sfermions}\tilde{f}^\dagger T^a\tilde{f}\right)\left(\sum_{Higgs} H^\dagger T^a H\right)\right] + \cdots, \quad (3.17)$$

   where $\cdots$ are the terms which do not contribute to sfermion masses. The factor 2 in (3.17) is the Newton's binomial coefficient. Since sfermion masses are our interest, therefore, in the second factor in (3.17), only the diagonal $T_3$ , $T_8$ and non-diagonal $T_4$ satisfy this purpose. Let us calculate the second factor in (3.17):

$$H_3 \equiv \sum_{H=\chi,\chi',\rho,\rho'} < H^\dagger > T_3 < H >= \frac{1}{4}(u^2 - u'^2) - \frac{1}{4}(v^2 - v'^2)$$
$$= -\frac{1}{4}\left(u^2\frac{\cos 2\beta}{s^2_\beta} + v^2\frac{\cos 2\gamma}{c^2_\gamma}\right), \quad (3.18)$$

$$H_8 \equiv \sum_{H=\chi,\chi',\rho,\rho'} < H^\dagger > T_8 < H >$$
$$= \frac{1}{2\sqrt{3}}\left\{\frac{1}{2}(u^2 - u'^2) + \frac{1}{2}(v^2 - v'^2) - (w^2 - w'^2)\right\}$$
$$= \frac{1}{4\sqrt{3}}\left[v^2\frac{\cos 2\gamma}{c^2_\gamma} - (u^2 - 2w^2)\frac{\cos 2\beta}{s^2_\beta}\right] \quad (3.19)$$

$$
\begin{aligned}
H_4 &\equiv \sum_{H=\chi,\chi',\rho,\rho'} < H^\dagger > T_4 < H > \\
&= \frac{1}{2}(uw - u'w') = -\frac{1}{2}uw\frac{\cos 2\beta}{s_\beta^2}.
\end{aligned} \tag{3.20}
$$

In (3.18), (3.19) and (3.20) we have used [18, 19]

$$
\tan\beta = \frac{u}{u'} = \frac{w}{w'}, \qquad \tan\gamma = \frac{v'}{v}. \tag{3.21}
$$

Here we have taken into account that for antitriplets, $T_a, a = 3, 8, 4$ changes a sign. Note that the contribution from $T_4$ is proportional to $u$ – the lepton number violating parameter.

Let us consider the first factor in (3.17). Since the singlet fields do not give contribution, hence for sleptons we have:

$$
SL_3 \equiv \tilde{L}^\dagger_{aL} T_3 \tilde{L}_{aL} = \frac{1}{2}\tilde{\nu}^*_{aL}\tilde{\nu}_{aL} - \frac{1}{2}\tilde{l}^*_{aL}\tilde{l}_{aL}, \tag{3.22}
$$

$$
SL_8 \equiv \tilde{L}^\dagger_{aL} T_8 \tilde{L}_{aL} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\tilde{\nu}^*_{aL}\tilde{\nu}_{aL} + \frac{1}{2}\tilde{l}^*_{aL}\tilde{l}_{aL} - \tilde{\nu}^{c*}_{aL}\tilde{\nu}^c_{aL}\right), \tag{3.23}
$$

$$
SL_4 \equiv \tilde{L}^\dagger_{aL} T_4 \tilde{L}_{aL} = \frac{1}{2}\tilde{\nu}^*_{aL}\tilde{\nu}^c_{aL} + \frac{1}{2}\tilde{\nu}^{c*}_{aL}\tilde{\nu}_{aL}. \tag{3.24}
$$

Analogously for squarks, the contributions from one triplet and two antitriplets are: from the first triplet

$$
SQ_3 \equiv \widetilde{Q}^\dagger_{1L} T_3 \widetilde{Q}_{1L} = \frac{1}{2}\tilde{u}^*_{1L}\tilde{u}_{1L} - \frac{1}{2}\tilde{d}^*_{1L}\tilde{d}_{1L}, \tag{3.25}
$$

$$
SQ_8 \equiv \widetilde{Q}^\dagger_{1L} T_8 \widetilde{Q}_{1L} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\tilde{u}^*_{1L}\tilde{u}_{1L} + \frac{1}{2}\tilde{d}^*_{1L}\tilde{d}_{1L} - \tilde{u}'^*_L\tilde{u}'_L\right), \tag{3.26}
$$

$$
SQ_4 \equiv \widetilde{Q}^\dagger_{1L} T_4 \widetilde{Q}_{1L} = \frac{1}{2}\tilde{u}^*_{1L}\tilde{u}'_L + \frac{1}{2}\tilde{u}'^*_L\tilde{u}_{1L}, \tag{3.27}
$$

from two antitriplets:

$$
SaQ_3 \equiv -\widetilde{Q}^\dagger_{\alpha L} T_3 \widetilde{Q}_{\alpha L} = -\frac{1}{2}\tilde{d}^*_{\alpha L}\tilde{d}_{\alpha L} + \frac{1}{2}\tilde{u}^*_{\alpha L}\tilde{u}_{\alpha L}, \tag{3.28}
$$

$$
SaQ_8 \equiv -\widetilde{Q}^\dagger_{\alpha L} T_8 \widetilde{Q}_{\alpha L} = -\frac{1}{2\sqrt{3}}\left(\tilde{u}^*_{\alpha L}\tilde{u}_{\alpha L} + \tilde{d}^*_{\alpha L}\tilde{d}_{\alpha L} - 2\tilde{d}'^*_{\alpha L}\tilde{d}'_{\alpha L}\right), \tag{3.29}
$$

$$
SaQ_4 \equiv -\widetilde{Q}^\dagger_{\alpha L} T_4 \widetilde{Q}_{\alpha L} = -\frac{1}{2}\tilde{d}^*_{\alpha L}\tilde{d}'_{\alpha L} - \frac{1}{2}\tilde{d}'^*_{\alpha L}\tilde{d}_{\alpha L}. \tag{3.30}
$$

Thus, the contribution from $SU(3)_L$ subgroup to slepton masses are:

$$
g^2(SL_3 \times H_3 + SL_8 \times H_8 + SL_4 \times H_4), \tag{3.31}
$$

and to squark masses:

$$
g^2[(SQ_3 + SaQ_3) \times H_3 + (SQ_8 + SaQ_8) \times H_8 + (SQ_4 + SaQ_4) \times H_4]. \tag{3.32}
$$

2. *D-term contribution from $U(1)_X$:*

First, for the Higgs part, we have

$$\begin{aligned} H_1 &\equiv \sum_{H=\chi,\chi',\rho,\rho'} < H^\dagger > X < H > \\ &= -\frac{1}{6}(u^2 - u'^2) + \frac{2}{6}(v^2 - v'^2) - \frac{1}{6}(w^2 - w'^2) \\ &= \frac{1}{6}\left[(u^2 + w^2)\frac{\cos 2\beta}{s_\beta^2} + 2v^2\frac{\cos 2\gamma}{c_\gamma^2}\right] \end{aligned} \tag{3.33}$$

Similarly, for sleptons

$$SL_1 \equiv -\frac{1}{3}(\tilde{\nu}^*_{aL}\tilde{\nu}_{aL} + \tilde{l}^*_{aL}\tilde{l}_{aL} + \tilde{\nu}^{c*}_{aL}\tilde{\nu}^c_{aL}) + \tilde{l}^{c*}_{aL}\tilde{l}^c_{aL}. \tag{3.34}$$

For squarks in the first generation we get:

$$SQ_1 \equiv \frac{1}{3}(\tilde{u}^*_{1L}\tilde{u}_{1L} + \tilde{d}^*_{1L}\tilde{d}_{1L} + \tilde{u}'^*_L\tilde{u}'_L) - \frac{2}{3}(\tilde{u}^{c*}_{1L}\tilde{u}^c_{1L} + \tilde{u}'^{c*}_L\tilde{u}'^c_L) + \frac{1}{3}\tilde{d}^{c*}_{1L}\tilde{d}^c_{1L}. \tag{3.35}$$

For squarks in the last two generations we get also:

$$SaQ_1 \equiv -\frac{2}{3}\tilde{u}^{c*}_{\alpha L}\tilde{u}^c_{\alpha L} + \frac{1}{3}(\tilde{d}^{c*}_{\alpha L}\tilde{d}^c_{\alpha L} + \tilde{d}'^{c*}_{\alpha L}\tilde{d}'^c_{\alpha L}). \tag{3.36}$$

The contribution from subgroup $U(1)_X$ to slepton masses is

$$g'^2 \times SL_1 \times H_1 = g^2t^2 \times SL_1 \times H_1 \tag{3.37}$$

where [22]

$$t^2 = (g'/g)^2 = \frac{3s_W^2}{3 - 4s_W^2} \tag{3.38}$$

and to squark masses:

$$g'^2 \times H_1(SQ_1 + SaQ_1) = g^2t^2 \times H_1(SQ_1 + SaQ_1). \tag{3.39}$$

The total contribution is a result of summation over two above mentioned subgroup parts. In contradiction to the MSSM, the contribution from $T_4$ is lepton number violating ($\Delta L = \pm 2$). We will deal with this in next section. It is easy to realize that *the D-term contributions are diagonal.*

## 4. Slepton masses

Relevant mass terms for sleptons arisen from the $F, D$-terms and the soft terms are as follows:

$$\begin{aligned} \mathcal{L}_{slepton} &= M^2_{ab}\widetilde{L}^*_{aL}\widetilde{L}_{bL} + m^2_{ab}\widetilde{l}^{c*}_{aL}\widetilde{l}^c_{bL} + \frac{1}{4}\mu_{0a}\mu_{0b}\tilde{L}^*_{aL}\tilde{L}_{bL} \\ &+ \Big[M'^2_a\chi^*\widetilde{L}_{aL} + \eta_{ab}\widetilde{L}_{aL}\rho'\widetilde{l}^c_{Lb} \end{aligned}$$

$$
\begin{aligned}
&+\upsilon_a\epsilon\widetilde{L}_{aL}\chi\rho+\varepsilon_{ab}\epsilon\widetilde{L}_{aL}\widetilde{L}_{bL}\rho \\
&+\frac{1}{4}\mu_{0a}\mu_\chi\chi^*\tilde{L}_{aL}+\frac{1}{6}\mu_\chi\lambda_a\epsilon\tilde{L}_{aL}\chi'^*\rho \\
&+\frac{1}{6}\mu_\rho\left(\lambda_a\epsilon\tilde{L}_{aL}\chi\rho'^*+\lambda'_{ab}\epsilon\tilde{L}_{aL}\tilde{L}_{bL}\rho'^*\right)+\frac{1}{6}\mu_\rho\rho^*\left(\gamma_{ab}\tilde{L}_{aL}\tilde{l}^c_{bL}\right) \\
&+\frac{1}{6}\mu_{0a}\left(\gamma_{ab}\chi'^*.\rho'\tilde{l}^c_{bL}+2\lambda'_{ab}\epsilon\chi'^*\tilde{L}_{bL}\rho\right)+\frac{1}{9}\gamma_{ab}\lambda_a\epsilon\rho'\chi^*\rho^*.\tilde{l}^c_{bL} \\
&+\frac{2}{9}\lambda'_{ab}\lambda_a[(\chi^*\tilde{L}_{bL})(\rho^*\rho)-(\rho^*\tilde{L}_{bL})(\chi^*\rho)]+H.c\Big] \\
&+\frac{1}{9}\gamma_{ab}\gamma_{ab'}\rho'^*\rho'\tilde{l}^c_{bL}\tilde{l}^{c*}_{b'L}+\frac{1}{9}\gamma_{ab}\gamma_{a'b}(\tilde{L}_{aL}\rho')(\tilde{L}_{a'L}\rho')^* \\
&+\frac{1}{9}\lambda_a\lambda_b[(\tilde{L}^*_{aL}\tilde{L}_{bL})(\rho^*\rho)-(\tilde{L}^*_{aL}\rho)(\rho^*\tilde{L}_{bL})] \\
&+\frac{1}{9}\lambda_a\lambda_b[(\tilde{L}^*_{aL}\tilde{L}_{bL})(\chi^*\chi)-(\tilde{L}^*_{aL}\chi)(\chi^*\tilde{L}_{bL})] \\
&+\frac{4}{9}\lambda'_{ca}\lambda'_{cb}[(\tilde{L}^*_{aL}\tilde{L}_{bL})(\rho^*\rho)-(\tilde{L}^*_{aL}\rho)(\rho^*\tilde{L}_{bL})] \\
&+g^2(SL_3\times H_3+SL_8\times H_8+SL_4\times H_4)+g^2t^2\times SL_1\times H_1. \qquad (4.1)
\end{aligned}
$$

Expanding the $D$-term contribution [in the last line of (4.1)] yields

$$
\begin{aligned}
D_L &\equiv g^2(SL_3\times H_3+SL_8\times H_8+SL_4\times H_4)+g^2t^2\times SL_1\times H_1 \\
&= g^2\left\{\tilde{\nu}^*_{aL}\tilde{\nu}_{aL}\left(\frac{1}{2}H_3+\frac{1}{2\sqrt{3}}H_8-\frac{t^2}{3}H_1\right)\right. \\
&\quad+\tilde{l}^*_{aL}\tilde{l}_{aL}\left(-\frac{1}{2}H_3+\frac{1}{2\sqrt{3}}H_8-\frac{t^2}{3}H_1\right) \\
&\quad\left.+\tilde{\nu}^{c*}_{aL}\tilde{\nu}^c_{aL}\left(-\frac{1}{\sqrt{3}}H_8-\frac{t^2}{3}H_1\right)+\tilde{l}^{c*}_{aL}\tilde{l}^c_{aL}t^2H_1+\frac{1}{2}(\tilde{\nu}^*_{aL}\tilde{\nu}^c_{aL}H_4+H.c.)\right\} \qquad (4.2)
\end{aligned}
$$

The terms containing mixture of scalar Higgs bosons among sleptons followed from $\mathcal{L}_{slepton}$ is:

$$
\begin{aligned}
\mathcal{L}_{mix} &= M'^2_a\chi^*\widetilde{L}_{aL}+\upsilon_a\epsilon\widetilde{L}_{aL}\chi\rho \\
&+\frac{1}{4}\mu_{0a}\mu_\chi\chi^*\tilde{L}_{aL}+\frac{1}{6}\mu_\chi\lambda_a\epsilon\tilde{L}_{aL}\chi'^*\rho \\
&+\frac{1}{6}\mu_\rho\lambda_a\epsilon\tilde{L}_{aL}\chi\rho'^*+\frac{1}{9}\gamma_{ab}\lambda_a\epsilon\rho'\chi^*\rho^*.\tilde{l}^c_{bL} \\
&+\frac{1}{6}\mu_{0a}\left(\gamma_{ab}\chi'^*.\rho'\tilde{l}^c_{bL}+2\lambda'_{ab}\epsilon\chi'^*\tilde{L}_{bL}\rho\right) \\
&+\frac{2}{9}\lambda'_{ab}\lambda_a[(\chi^*\tilde{L}_{bL})(\rho^*\rho)-(\rho^*\tilde{L}_{bL})(\chi^*\rho)]+H.c. \\
&=\left(M'^2_a+\frac{1}{4}\mu_{0a}\mu_\chi\right)(\chi^{0*}_1\widetilde{\nu}_{aL}+\chi^+\widetilde{l}_{aL}+\chi^{0*}_2\widetilde{\nu}^c_{aL}) \\
&+\upsilon_a[(-\widetilde{\nu}_{aL}\chi^0_2+\widetilde{\nu}^c_{aL}\chi^0_1)\rho^0+\widetilde{l}_{aL}(-\chi^0_1\rho^+_2+\chi^0_2\rho^+_1)] \\
&+\left(\frac{1}{6}\mu_\chi\lambda_a-\frac{1}{3}\mu_{0b}\lambda'_{ba}\right)[(-\widetilde{\nu}_{aL}\chi'^{0*}_2+\widetilde{\nu}^c_{aL}\chi'^{0*}_1)\rho^0+\widetilde{l}_{aL}(-\chi'^{0*}_1\rho^+_2+\chi'^{0*}_2\rho^+_1)] \\
&+\frac{1}{6}\mu_\rho\lambda_a[(-\widetilde{\nu}_{aL}\chi^0_2+\widetilde{\nu}^c_{aL}\chi^0_1)\rho'^{0*}+\widetilde{l}_{aL}(-\chi^0_1\rho'^+_2+\chi^0_2\rho'^+_1)]
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{9}\gamma_{ab}\lambda_a[(-\rho_1'^-\chi_2^{0*}+\rho_2'^-\chi_1^{0*})\rho^{0*}+\rho'^0(\chi_2^{0*}\rho_1^- - \chi_1^{0*}\rho_2^-)]\tilde{l}_{bL}^c \\
&+\frac{1}{6}\mu_{0a}\gamma_{ab}(\chi_1'^{0*}\rho_1'^- + \chi'^-.\rho'^0 + \chi_2'^{0*}\rho_2'^-)\tilde{l}_{bL}^c \\
&+\frac{2}{9}\lambda'_{ab}\lambda_a[(\chi_1^{0*}\tilde{\nu}_{bL}+\chi^+\tilde{l}_{bL}+\chi_2^{0*}\tilde{\nu}_{bL}^c)(\rho^{0*}\rho^0) \\
&-\rho^{0*}\tilde{l}_{bL}(\chi_1^{0*}\rho_1^+ + \chi^+\rho^0 + \chi_2^{0*}\rho_2^+)] + H.c. \qquad (4.3)
\end{aligned}
$$

Now we have to expand neutral Higgs fields around the VEVs as

$$
\begin{aligned}
\chi^T &= \left(\tfrac{u+S_1+iA_1}{\sqrt{2}},\, \chi^-,\, \tfrac{w+S_2+iA_2}{\sqrt{2}}\right), \qquad \rho^T = \left(\rho_1^+,\, \tfrac{v+S_5+iA_5}{\sqrt{2}},\, \rho_2^+\right), \\
\chi'^T &= \left(\tfrac{u'+S_3+iA_3}{\sqrt{2}},\, \chi'^+,\, \tfrac{w'+S_4+iA_4}{\sqrt{2}}\right), \qquad \rho'^T = \left(\rho_1'^-,\, \tfrac{v'+S_6+iA_6}{\sqrt{2}},\, \rho_2'^-\right). \qquad (4.4)
\end{aligned}
$$

where for short, the neutral scalar is expressed through the VEV and physical field (in breve) as follows

$$
h^0 = \frac{1}{\sqrt{2}}(vev + \breve{h}). \qquad (4.5)
$$

From Eq. (4.3) we see that, there is mixing among charged Higgs boson $\chi'^-$ with $\tilde{l}_{bL}^c$ as well as neutral Higgs fields $\breve{\chi}_1'^0$ with neutral sleptons such as $\tilde{\nu}_{bL}^c$, etc. To remove this mixing, we have to impose $R$-parity condition.

Imposing $R$-parity conservation on (4.3) yields

$$
\begin{aligned}
&M_a'^2 + \frac{1}{4}\mu_{0a}\mu_\chi = 0, \quad \upsilon_a = 0, \quad \mu_\rho\lambda_a = 0, \qquad (4.6) \\
&\frac{1}{6}\mu_\chi\lambda_a - \frac{1}{3}\mu_{0b}\lambda'_{ba} = 0, \quad \gamma_{ab}\lambda_a = 0, \quad \lambda'_{ab}\lambda_a = 0, \qquad (4.7) \\
&\mu_{0a}\gamma_{ab} = 0. \qquad (4.8)
\end{aligned}
$$

Note that the conditions in (4.6)–(4.8) contain also the constraint equations at the tree level for $\tilde{\nu}_{aL}$ and $\tilde{\nu}_{aL}^c$.

Taking into account of (4.6)–(4.8), the slepton mass Lagrangian becomes

$$
\begin{aligned}
\mathcal{L}_{slepton} &= D_L + \left(M_{ab}^2 + \frac{1}{4}\mu_{0a}\mu_{0b}\right)(\tilde{\nu}_{aL}^*\tilde{\nu}_{bL} + \tilde{l}_{aL}^*\tilde{l}_{bL} + \tilde{\nu}_{aL}^{c*}\tilde{\nu}_{bL}^c) + m_{ab}^2\tilde{l}_{aL}^{c*}\tilde{l}_{bL}^c \\
&+ \left\{\frac{1}{\sqrt{2}}\left(\eta_{ab}v' + \frac{1}{6}\mu_\rho\gamma_{ab}v\right)\tilde{l}_{aL}\tilde{l}_{bL}^c - \frac{1}{\sqrt{2}}\varepsilon_{ab}v(\tilde{\nu}_{aL}\tilde{\nu}_{bL}^c - \tilde{\nu}_{bL}\tilde{\nu}_{aL}^c)\right. \\
&\left. - \frac{1}{6\sqrt{2}}\mu_\rho\lambda'_{ab}v'(\tilde{\nu}_{aL}\tilde{\nu}_{bL}^c - \tilde{\nu}_{bL}\tilde{\nu}_{aL}^c) + H.c.\right\} \\
&+\frac{1}{9}\gamma_{ab}\gamma_{ab'}\tilde{l}_{bL}^{c*}\tilde{l}_{b'L}^c\frac{v'^2}{2} + \frac{1}{9}\gamma_{ab}\gamma_{a'b}(\tilde{l}_{aL}\tilde{l}_{a'L}^\dagger)\frac{v'^2}{2} \\
&+\frac{1}{18}v^2(\lambda_a\lambda_b + 4\lambda'_{ca}\lambda'_{cb})(\tilde{\nu}_{aL}^*\tilde{\nu}_{bL} + \tilde{\nu}_{aL}^{c*}\tilde{\nu}_{bL}^c) \\
&+\frac{1}{18}\lambda_a\lambda_b\left[w^2\tilde{\nu}_{aL}^*\tilde{\nu}_{bL} + u^2\tilde{\nu}_{aL}^{c*}\tilde{\nu}_{bL}^c\right. \\
&\left.+(u^2+w^2)\tilde{l}_{aL}^*\tilde{l}_{bL} - uw(\tilde{\nu}_{aL}^*\tilde{\nu}_{bL}^c + \tilde{\nu}_{aL}^{c*}\tilde{\nu}_{bL})\right]. \qquad (4.9)
\end{aligned}
$$

### 4.1 Charged sleptons

From Eq. (4.9), the mass Lagrangian for charged sleptons is given by

$$\begin{aligned}
\mathcal{L}_{Charlepton} &= \left[M^2_{ab} + \frac{1}{4}\mu_{0a}\mu_{0b} + \frac{v'^2}{18}\gamma_{ca}\gamma_{cb} + \frac{g^2}{2}\delta_{ab}\left(-H_3 + \frac{1}{\sqrt{3}}H_8 - \frac{2t^2}{3}H_1\right)\right]\tilde{l}^*_{aL}\tilde{l}_{bL} \\
&+ \left(m^2_{ab} + \frac{v'^2}{18}\gamma_{ca}\gamma_{cb} + g^2t^2H_1\delta_{ab}\right)\tilde{l}^{c*}_{aL}\tilde{l}^c_{bL} \\
&+ \left[\frac{1}{\sqrt{2}}\left(\eta_{ab}v' + \frac{1}{6}\mu_\rho\gamma_{ab}v\right)\tilde{l}_{aL}\tilde{l}^c_{bL} + H.c.\right] \\
&+ \frac{1}{18}\lambda_a\lambda_b(u^2+w^2)\tilde{l}^*_{aL}\tilde{l}_{bL} \qquad (4.10)
\end{aligned}$$

For analysis below, let us denote

$$\begin{aligned}
B_{ab} &= M^2_{ab} + \frac{1}{4}\mu_{0a}\mu_{0b} + \frac{v'^2}{18}\gamma_{ca}\gamma_{cb} + \frac{1}{18}\lambda_a\lambda_b(u^2+w^2) \\
&+ \frac{g^2}{2}\delta_{ab}\left(-H_3 + \frac{1}{\sqrt{3}}H_8 - \frac{2t^2}{3}H_1\right), \qquad (4.11)\\
C_{ab} &= m^2_{ab} + \frac{v'^2}{18}\gamma_{ca}\gamma_{cb} + g^2t^2H_1\delta_{ab}, \qquad (4.12)\\
D_{ab} &= \frac{1}{\sqrt{2}}\left(\eta_{ab}v' + \frac{1}{6}\mu_\rho\gamma_{ab}v\right). \qquad (4.13)
\end{aligned}$$

For the sake of convenience, let us denote $\tilde{l}^{c*}_{aL} \equiv \tilde{l}_{aR}$. Then, in the base $(\tilde{l}_{aL}, \tilde{l}_{bR}) = (\tilde{l}_{1L}$, $\tilde{l}_{2L}$, $\tilde{l}_{3L}$, $\tilde{l}_{1R}$, $\tilde{l}_{2R}$, $\tilde{l}_{3R})$, the mass matrix is given by

$$\begin{pmatrix} B_{ab} & D_{ab} \\ D_{ab} & C_{ab} \end{pmatrix}. \qquad (4.14)$$

To deal with this $6 \times 6$ matrix, following Ref.[14], we assume that there is substantial mixing among $(\tilde{\tau}_L, \tilde{\tau}_R)$ *only*. Hereafter, we adopt $\tilde{\tau} = \tilde{l}_3$, $\tilde{t} = \tilde{u}_3$, $\tilde{b} = \tilde{d}_3$, etc. This means that, non-vanishing matrix elements in (4.14) are $B_{11}$, $B_{22}$, $B_{33}$, $C_{11}$, $C_{22}$, $C_{33}$, $D_{33}$.

Diagonalizing the above matrix, we get eigenmasses and eigenstates given in Table 1. and two others are

**Table 1:** Masses and eigenstates of charged sleptons

| Eigenstate | $\tilde{l}_{1L}$ | $\tilde{l}_{2L}$ | $\tilde{l}_{1R}$ | $\tilde{l}_{2R}$ |
|---|---|---|---|---|
| $(\text{Mass})^2$ | $B_{11}$ | $B_{22}$ | $C_{11}$ | $C_{22}$ |

$$\begin{aligned}
\tilde{\tau}_L &= s_{\theta_s}\tilde{l}_{3R} - c_{\theta_s}\tilde{l}_{3L}, \qquad (4.15)\\
\tilde{\tau}_R &= c_{\theta_s}\tilde{l}_{3R} + s_{\theta_s}\tilde{l}_{3L}, \qquad (4.16)
\end{aligned}$$

with respective masses

$$m^2_{\tilde{\tau}_L} = \frac{1}{2}(B_{33} + C_{33} - \Delta), \quad (4.17)$$

$$m^2_{\tilde{\tau}_R} = \frac{1}{2}(B_{33} + C_{33} + \Delta), \quad (4.18)$$

where

$$\Delta = \sqrt{(C_{33} - B_{33})^2 + 4D^2_{33}}, \quad (4.19)$$

$$t_{2\theta_s} = \frac{2D_{33}}{C_{33} - B_{33}}. \quad (4.20)$$

With the mentioned assumption, we have

$$m^2_{\tilde{l}_{1L}} = M^2_{11} + \frac{1}{4}\mu^2_{01} + \frac{v'^2}{18}\gamma^2_{c1} + \frac{1}{18}\lambda^2_1(u^2 + w^2) - \frac{g^2}{2}\left(H_3 - \frac{1}{\sqrt{3}}H_8 + \frac{2t^2}{3}H_1\right), \quad (4.21)$$

$$m^2_{\tilde{l}_{2L}} = M^2_{22} + \frac{1}{4}\mu^2_{02} + \frac{v'^2}{18}\gamma^2_{c2} + \frac{1}{18}\lambda^2_2(u^2 + w^2) - \frac{g^2}{2}\left(H_3 - \frac{1}{\sqrt{3}}H_8 + \frac{2t^2}{3}H_1\right), \quad (4.22)$$

$$m^2_{\tilde{l}_{1R}} = m^2_{11} + \frac{v'^2}{18}\gamma^2_{c1} + g^2t^2H_1, \quad (4.23)$$

$$m^2_{\tilde{l}_{2R}} = m^2_{22} + \frac{v'^2}{18}\gamma^2_{c2} + g^2t^2H_1, \quad (4.24)$$

$$(4.25)$$

For the highest sleptons - staus:

$$m^2_{\tilde{\tau}_L} = \frac{1}{2}\Big[M^2_{33} + m^2_{33} + \frac{v'^2}{9}\gamma^2_{c3} + \frac{1}{4}\mu^2_{03} + \frac{1}{18}\lambda^2_3(u^2 + w^2) - \frac{g^2}{2}\left(H_3 - \frac{1}{\sqrt{3}}H_8 - \frac{4t^2}{3}H_1\right) - \Delta\Big], \quad (4.26)$$

$$m^2_{\tilde{\tau}_R} = \frac{1}{2}\Big[M^2_{33} + m^2_{33} + \frac{v'^2}{9}\gamma^2_{c3} + \frac{1}{4}\mu^2_{03} + \frac{1}{18}\lambda^2_3(u^2 + w^2) - \frac{g^2}{2}\left(H_3 - \frac{1}{\sqrt{3}}H_8 - \frac{4t^2}{3}H_1\right) + \Delta\Big], \quad (4.27)$$

with

$$\Delta = \left\{\Big[M^2_{33} + \frac{1}{4}\mu^2_{03} + \frac{1}{18}\lambda^2_3(u^2 + w^2) - m^2_{33} - \frac{g^2}{2}\left(H_3 - \frac{1}{\sqrt{3}}H_8 + \frac{8t^2}{3}H_1\right)\Big]^2 + 2\left(\eta_{33}v' + \frac{1}{6}\mu_\rho\gamma_{33}v\right)^2\right\}^{\frac{1}{2}} \quad (4.28)$$

### 4.2 Sneutrinos

Eq. (4.9) provides the following mass Lagrangian for sneutrinos:

$$\mathcal{L}_{sneutrinos} = \left[\frac{g^2}{2}\delta_{ab}\left(H_3 + \frac{1}{\sqrt{3}}H_8 - \frac{2t^2}{3}H_1\right) + M^2_{ab} + \frac{1}{4}\mu_{0a}\mu_{0b}\right]\tilde{\nu}^*_{aL}\tilde{\nu}_{bL}$$

$$
\begin{aligned}
&+\left[-g^2\delta_{ab}\left(\frac{1}{\sqrt{3}}H_8+\frac{t^2}{3}H_1\right)+M^2_{ab}+\frac{1}{4}\mu_{0a}\mu_{0b}\right]\tilde{\nu}^{c*}_{aL}\tilde{\nu}^c_{bL}\\
&-\left[\frac{1}{\sqrt{2}}\varepsilon_{ab}v(\tilde{\nu}_{aL}\tilde{\nu}^c_{bL}-\tilde{\nu}_{bL}\tilde{\nu}^c_{aL})\right.\\
&\left.+\frac{1}{6\sqrt{2}}\mu_\rho\lambda'_{ab}v'(\tilde{\nu}_{aL}\tilde{\nu}^c_{bL}-\tilde{\nu}_{bL}\tilde{\nu}^c_{aL})+H.c.\right]\\
&+\frac{1}{18}v^2(\lambda_a\lambda_b+4\lambda'_{ca}\lambda'_{cb})(\tilde{\nu}^*_{aL}\tilde{\nu}_{bL}+\tilde{\nu}^{c*}_{aL}\tilde{\nu}^c_{bL})\\
&+\frac{1}{18}\lambda_a\lambda_b\left[w^2\tilde{\nu}^*_{aL}\tilde{\nu}_{bL}+u^2\tilde{\nu}^{c*}_{aL}\tilde{\nu}^c_{bL}-uw(\tilde{\nu}^*_{aL}\tilde{\nu}^c_{bL}+\tilde{\nu}^{c*}_{aL}\tilde{\nu}_{bL})\right]\\
&-\frac{g^2}{4}uw\frac{cos2\beta}{s^2_\beta}(\tilde{\nu}^*_{aL}\tilde{\nu}^c_{aL}+H.c.)\\
=&\left[\frac{g^2}{2}\delta_{ab}\left(H_3+\frac{1}{\sqrt{3}}H_8-\frac{2t^2}{3}H_1\right)+M^2_{ab}+\frac{1}{4}\mu_{0a}\mu_{0b}\right.\\
&\left.+\frac{1}{18}v^2(\lambda_a\lambda_b+4\lambda'_{ca}\lambda'_{cb})+\frac{1}{18}\lambda_a\lambda_b w^2\right]\tilde{\nu}^*_{aL}\tilde{\nu}_{bL}\\
&+\left[-g^2\delta_{ab}\left(\frac{1}{\sqrt{3}}H_8+\frac{t^2}{3}H_1\right)+M^2_{ab}+\frac{1}{4}\mu_{0a}\mu_{0b}\right.\\
&\left.+\frac{1}{18}v^2(\lambda_a\lambda_b+4\lambda'_{ca}\lambda'_{cb})+\frac{1}{18}\lambda_a\lambda_b u^2\right]\tilde{\nu}^{c*}_{aL}\tilde{\nu}^c_{bL}\\
&-\left[\left(\sqrt{2}\varepsilon_{ab}v+\frac{\sqrt{2}}{6}\mu_\rho\lambda'_{ab}v'\right)\tilde{\nu}_{aL}\tilde{\nu}^c_{bL}\right.\\
&\left.-\frac{1}{2}uw\left(\frac{\lambda_a\lambda_b}{9}+\frac{g^2}{2}\frac{cos2\beta}{s^2_\beta}\right)\tilde{\nu}^*_{aL}\tilde{\nu}^c_{bL}+H.c.\right].
\end{aligned}
\tag{4.29}
$$

It is to be noticed that the last term in (4.29) is the mass-like (in the second order of fields) lepton-number violating ($\Delta L = \pm 2$). It is similar to the neutrino Majorona mass term; and this is a special feature of the supersymmetric version.

For the sake of convenience, we will use the following notation

$$
\begin{aligned}
A_{ab}=&\frac{g^2}{2}\delta_{ab}\left(H_3+\frac{1}{\sqrt{3}}H_8-\frac{2t^2}{3}H_1\right)+M^2_{ab}+\frac{1}{4}\mu_{0a}\mu_{0b}\\
&+\frac{1}{18}v^2(\lambda_a\lambda_b+4\lambda'_{ca}\lambda'_{cb})+\frac{1}{18}\lambda_a\lambda_b w^2,
\end{aligned}
\tag{4.30}
$$

$$
\begin{aligned}
G_{ab}=&-g^2\delta_{ab}\left(\frac{1}{\sqrt{3}}H_8+\frac{t^2}{3}H_1\right)+M^2_{ab}+\frac{1}{4}\mu_{0a}\mu_{0b}\\
&+\frac{1}{18}v^2(\lambda_a\lambda_b+4\lambda'_{ca}\lambda'_{cb})+\frac{1}{18}\lambda_a\lambda_b u^2,
\end{aligned}
\tag{4.31}
$$

$$
E_{ab}=-\sqrt{2}\left(\varepsilon_{ab}v+\frac{1}{6}\mu_\rho\lambda'_{ab}v'\right).
\tag{4.32}
$$

In the base $(\tilde{\nu}_{aL},\tilde{\nu}_{bR})=(\tilde{\nu}_{1L},\tilde{\nu}_{2L},\tilde{\nu}_{3L},\tilde{\nu}_{1R},\tilde{\nu}_{2R},\tilde{\nu}_{3R})$, the mass matrix is given by

$$
\begin{pmatrix} A_{ab} & E_{ab}\\ E_{ab} & G_{ab}\end{pmatrix}.
\tag{4.33}
$$

Eigenstates and eigenmasses in this case are completely analogous to the charged sleptons with replacements: $B_{33} \Rightarrow A_{33}$, $C_{33} \Rightarrow G_{33}$ and $D_{33} \Rightarrow E_{33}$.

As before, ignoring mixing among sneutrinos of two first generations, we get eigenmasses and eigenstates given in Table 2. and two other sneutrinos are

**Table 2:** Masses and eigenstates of charged sleptons

| Eigenstate | $\tilde{\nu}_{1L}$ | $\tilde{\nu}_{2L}$ | $\tilde{\nu}_{1R}$ | $\tilde{\nu}_{2R}$ |
|---|---|---|---|---|
| $(\mathrm{Mass})^2$ | $A_{11}$ | $A_{22}$ | $G_{11}$ | $G_{22}$ |

$$\tilde{\nu}_{\tau L} = s_{\theta_n}\tilde{\nu}_{3R} - c_{\theta_n}\tilde{\nu}_{3L}, \tag{4.34}$$

$$\tilde{\nu}_{\tau R} = c_{\theta_n}\tilde{\nu}_{3R} + s_{\theta_n}\tilde{\nu}_{3L}, \tag{4.35}$$

with respective masses

$$m^2_{\tilde{\nu}_{\tau L}} = \frac{1}{2}(A_{33} + G_{33} - \Delta_n), \tag{4.36}$$

$$m^2_{\tilde{\nu}_{\tau R}} = \frac{1}{2}(A_{33} + G_{33} + \Delta_n), \tag{4.37}$$

where

$$\Delta_n = \sqrt{(G_{33} - A_{33})^2 + 4E^2_{33}}, \tag{4.38}$$

$$t_{2\theta_n} = \frac{2E_{33}}{G_{33} - A_{33}}. \tag{4.39}$$

With the mentioned assumption, we have

$$\begin{aligned} m^2_{\tilde{\nu}_{1L}} &= M^2_{11} + \frac{1}{4}\mu^2_{01} + \frac{g^2}{2}\left(H_3 + \frac{1}{\sqrt{3}}H_8 - \frac{2t^2}{3}H_1\right) \\ &\quad + \frac{1}{18}v^2(\lambda^2_1 + 4\lambda'^2_{c1}) + \frac{1}{18}\lambda^2_1 w^2, \end{aligned} \tag{4.40}$$

$$\begin{aligned} m^2_{\tilde{\nu}_{2L}} &= M^2_{22} + \frac{1}{4}\mu^2_{02} + \frac{g^2}{2}\left(H_3 + \frac{1}{\sqrt{3}}H_8 - \frac{2t^2}{3}H_1\right) \\ &\quad + \frac{1}{18}v^2(\lambda^2_2 + 4\lambda'^2_{c2}) + \frac{1}{18}\lambda^2_2 w^2, \end{aligned} \tag{4.41}$$

$$\begin{aligned} m^2_{\tilde{\nu}_{1R}} &= M^2_{11} + \frac{1}{4}\mu^2_{01} - g^2\left(\frac{1}{\sqrt{3}}H_8 + \frac{t^2}{3}H_1\right) \\ &\quad + \frac{1}{18}v^2(\lambda^2_1 + 4\lambda'^2_{c1}) + \frac{1}{18}\lambda^2_1 u^2, \end{aligned} \tag{4.42}$$

$$\begin{aligned} m^2_{\tilde{\nu}_{2R}} &= M^2_{22} + \frac{1}{4}\mu^2_{02} - g^2\left(\frac{1}{\sqrt{3}}H_8 + \frac{t^2}{3}H_1\right) \\ &\quad + \frac{1}{18}v^2(\lambda^2_2 + 4\lambda'^2_{c2}) + \frac{1}{18}\lambda^2_2 u^2. \end{aligned} \tag{4.43}$$

For the highest (tau) sneutrinos:

$$\begin{aligned} m^2_{\tilde{\nu}_{\tau L}} &= \frac{1}{2}\left[2M^2_{33} + \frac{1}{2}\mu^2_{03} + \frac{1}{9}v^2(\lambda^2_3 + 4\lambda'^3_{c3}) + \frac{1}{18}\lambda^2_3(w^2+u^2) \right. \\ &\quad \left. + \frac{g^2}{2}\left(H_3 - \frac{1}{\sqrt{3}}H_8 - \frac{4t^2}{3}H_1\right) - \Delta_n\right], \end{aligned} \tag{4.44}$$

$$\begin{aligned} m^2_{\tilde{\nu}_{\tau R}} &= \frac{1}{2}\left[2M^2_{33} + \frac{1}{2}\mu^2_{03} + \frac{1}{9}v^2(\lambda^2_3 + 4\lambda'^3_{c3}) + \frac{1}{18}\lambda^2_3(w^2+u^2) \right. \\ &\quad \left. + \frac{g^2}{2}\left(H_3 - \frac{1}{\sqrt{3}}H_8 - \frac{4t^2}{3}H_1\right) + \Delta_n\right], \end{aligned} \tag{4.45}$$

with

$$\Delta_n = \sqrt{\left[\frac{g^2}{2}\left(H_3 + \sqrt{3}H_8\right) + \frac{1}{18}\lambda^2_3(w^2-u^2)\right]^2 + 8\left(\varepsilon_{33}v + \frac{1}{6}\mu_\rho\lambda'_{33}v'\right)^2}. \tag{4.46}$$

Comparing (4.21), (4.22) with (4.40), (4.41), we see that, besides $\not{R}$ – coefficients, without $D$-term contribution, there are degeneracies among left-handed and right-handed sneutrinos, namely, $\tilde{\nu}_{1L}$ among $\tilde{\nu}_{1R}$ and $\tilde{\nu}_{2L}$ among $\tilde{\nu}_{2R}$.

## 5. Squark masses

Due to the fact that the exotic quarks in the model under consideration have electric charges equal to that of the ordinary ones, squark mass mixing matrices are expected to be larger than $6\times 6$.

### 5.1 Squark mass Lagrangian

As before, the $D$-term contribution is diagonal, and let us denote it by $D_Q$:

$$\begin{aligned} D_Q &\equiv g^2[(SQ_3 + SaQ_3)\times H_3 + (SQ_8 + SaQ_8)\times H_8 + (SQ_4 + SaQ_4)\times H_4] \\ &\quad + g^2t^2 \times H_1(SQ_1 + SaQ_1) \\ &= g^2\left\{\tilde{u}^*_{1L}\tilde{u}_{1L}\left(\frac{1}{2}H_3 + \frac{1}{2\sqrt{3}}H_8 + \frac{t^2}{3}H_1\right) - \frac{2t^2}{3}H_1\tilde{u}^{c*}_{1L}\tilde{u}^c_{1L}\right. \\ &\quad + \tilde{d}^*_{1L}\tilde{d}_{1L}\left(-\frac{1}{2}H_3 + \frac{1}{2\sqrt{3}}H_8 + \frac{t^2}{3}H_1\right) + \frac{t^2}{3}H_1\tilde{d}^{c*}_{1L}\tilde{d}^c_{1L} \\ &\quad + \tilde{u}'^*_L\tilde{u}'_L\left(-\frac{1}{\sqrt{3}}H_8 + \frac{t^2}{3}H_1\right) - \frac{2t^2}{3}H_1\tilde{u}'^{c*}_L\tilde{u}'^c_L \\ &\quad + \tilde{u}^*_{\alpha L}\tilde{u}_{\alpha L}\left(\frac{1}{2}H_3 - \frac{1}{2\sqrt{3}}H_8\right) - \frac{2t^2}{3}H_1\tilde{u}^{c*}_{\alpha L}\tilde{u}^c_{\alpha L} \\ &\quad - \tilde{d}^*_{\alpha L}\tilde{d}_{\alpha L}\left(\frac{1}{2}H_3 + \frac{1}{2\sqrt{3}}H_8\right) + \frac{t^2}{3}H_1\tilde{d}^{c*}_{\alpha L}\tilde{d}^c_{\alpha L} \\ &\quad \left. + \frac{1}{\sqrt{3}}H_8\tilde{d}'^*_{\alpha L}\tilde{d}'_{\alpha L} + \frac{t^2}{3}H_1\tilde{d}'^{c*}_{\alpha L}\tilde{d}'^c_{\alpha L} + \frac{1}{2}H_4\left(\tilde{u}^*_{1L}\tilde{u}'_L - \tilde{d}^*_{\alpha L}\tilde{d}'_{\alpha L} + H.c.\right)\right\}. \end{aligned} \tag{5.1}$$

Relevant mass terms for squarks arisen from the $F, D$ -terms and the soft terms are given by

$$
\begin{aligned}
\mathcal{L}_{squarks} = {} & D_Q + m^2_{Q1L}\widetilde{Q}^\dagger_{1L}\widetilde{Q}_{1L} + m^2_{Q\alpha\beta L}\widetilde{Q}^\dagger_{\alpha L}\widetilde{Q}_{\beta L} \\
& + m^2_{u_{ij}}\widetilde{u}^{c*}_{iL}\widetilde{u}^c_{jL} + m^2_{d_{ij}}\widetilde{d}^{c*}_{iL}\widetilde{d}^c_{jL} + m^2_{u'}\widetilde{u'}^{c*}_L\widetilde{u'}^c_L + m^2_{d'\alpha\beta}\widetilde{d'}^{c*}_{\alpha L}\widetilde{d'}^c_{\beta L} \\
& + \Big[ p_i\widetilde{Q}_{1L}\chi'\widetilde{u}^c_{iL} + p\widetilde{Q}_{1L}\chi'\widetilde{u'}^c_L + p_{\alpha i}\widetilde{Q}_{\alpha L}\rho\widetilde{u}^c_{iL} + r_\alpha\widetilde{Q}_{\alpha L}\rho\widetilde{u'}^c_L + h_i\widetilde{Q}_{1L}\rho'\widetilde{d}^c_{iL} \\
& + h'_\alpha\widetilde{Q}_{1L}\rho'\widetilde{d'}^c_{\alpha L} + h_{\alpha i}\widetilde{Q}_{\alpha L}\chi\widetilde{d}^c_{iL} + h'_{\alpha\beta}\widetilde{Q}_{\alpha L}\chi\widetilde{d'}^c_{\beta L} \\
& + \frac{1}{6}\mu_\chi\chi^*(\kappa_i\tilde{Q}_{1L}\tilde{u}^c_{iL} + \kappa'\tilde{Q}_{1L}\tilde{u}'^c_L) \\
& + \frac{1}{6}\mu_\chi\left(\Pi_{\alpha i}\chi'^*.\tilde{Q}_{\alpha L}\tilde{d}^c_{iL} + \Pi'_{\alpha\beta}\chi'^*.\tilde{Q}_{\alpha L}\tilde{d}'^c_{\beta L}\right) \\
& + \frac{1}{6}\mu_\rho\left(\pi_{\alpha i}\rho'^*.\tilde{Q}_{\alpha L}\tilde{u}^c_{iL} + \pi'_\alpha\rho'^*.\tilde{Q}_{\alpha L}\tilde{u}'^c_L\right) \\
& + \frac{1}{6}\mu_\rho\rho^*\left(\vartheta_i\tilde{Q}_{1L}\tilde{d}^c_{iL} + \vartheta'_\alpha\tilde{Q}_{1L}\tilde{d}'^c_{\alpha L}\right) \\
& + \frac{1}{6}\mu_{0a}\chi'^{\sigma *}(\xi_{a\alpha j}\tilde{Q}_{\alpha L\sigma}\tilde{d}^c_{jL} + \xi'_{a\alpha\beta}\tilde{Q}_{\alpha L\sigma}\tilde{d}'^c_{\beta L}) \\
& + \frac{1}{9}\lambda_a(\xi_{a\alpha i}\epsilon\tilde{Q}_{\alpha L}\chi^*\rho^*.\tilde{d}^c_{iL} + \xi'_{a\alpha\beta}\epsilon\tilde{Q}_{\alpha L}\chi^*\rho^*.\tilde{d}'^c_{\beta L}) + H.c.\Big] \\
& + \frac{1}{18}\big[(w'^2 + u'^2)(\kappa_i\kappa_j\tilde{u}^{c*}_{iL}\tilde{u}^c_{jL} + \kappa'^2\tilde{u}'^{c*}_L\tilde{u}'^c_L + \kappa_i\kappa'\tilde{u}^{c*}_{iL}\tilde{u}'^c_L + \kappa_i\kappa'\tilde{u}^c_{iL}\tilde{u}'^{c*}_L) \\
& + v'^2(\vartheta_i\vartheta_j\tilde{d}^{c*}_{iL}\tilde{d}^c_{jL} + \vartheta'_\alpha\vartheta'_\beta\tilde{d}'^{c*}_{\alpha L}\tilde{d}'^c_{\beta L} + \vartheta_i\vartheta'_\alpha\tilde{d}^{c*}_{iL}\tilde{d}'^c_{\alpha L} + \vartheta_i\vartheta'_\alpha\tilde{d}^c_{iL}\tilde{d}'^{c*}_{\alpha L})\big] \\
& + \frac{1}{18}\big[v^2(\pi_{\alpha i}\pi_{\alpha j}\tilde{u}^{c*}_{iL}\tilde{u}^c_{jL} + \pi'^2_\alpha\tilde{u}'^{c*}_L\tilde{u}'^c_L + \pi_{\alpha i}\pi'_\alpha\tilde{u}^{c*}_{iL}\tilde{u}'^c_L + \pi_{\alpha i}\pi'_\alpha\tilde{u}^c_{iL}\tilde{u}'^{c*}_L) \\
& + (w^2 + u^2)(\Pi_{\alpha i}\Pi_{\alpha j}\tilde{d}^{c*}_{iL}\tilde{d}^c_{jL} + \Pi'_{\alpha\beta}\Pi'_{\alpha\delta}\tilde{d}'^{c*}_{\beta L}\tilde{d}'^c_{\delta L} \\
& + \Pi_{\alpha i}\Pi'_{\alpha\beta}\tilde{d}^{c*}_{iL}\tilde{d}'^c_{\beta L} + \Pi_{\alpha i}\Pi'_{\alpha\beta}\tilde{d}^c_{iL}\tilde{d}'^{c*}_{\beta L})\big] \\
& + \frac{1}{18}\big[u'^2\kappa_i^2\widetilde{u}^*_{1L}\widetilde{u}_{1L} + w'^2\kappa_i^2\widetilde{u}'^*_L\widetilde{u}'_L + v^2\pi_{\alpha i}\pi_{\beta i}\widetilde{u}_{\alpha L}\widetilde{u}^*_{\beta L} \\
& + (u'w'\kappa_i^2\widetilde{u}^*_{1L}\widetilde{u'}_L - u'v\kappa_i\pi_{\alpha i}\widetilde{u}^*_{1L}\widetilde{u}_{\alpha L} - w'v\kappa_i\pi_{\alpha i}\widetilde{u}'^*_L\widetilde{u}_{\alpha L} + H.c.)\big] \\
& + \frac{1}{18}\big[u'^2\kappa'^2\widetilde{u}^*_{1L}\widetilde{u}_{1L} + w'^2\kappa'^2\widetilde{u}'^*_L\widetilde{u}'_L + v^2\pi'_\alpha\pi'_\beta\widetilde{u}_{\alpha L}\widetilde{u}^*_{\beta L} \\
& + (u'w'\kappa'^2\widetilde{u}^*_{1L}\widetilde{u'}_L - u'v\kappa'\pi'_\alpha\widetilde{u}^*_{1L}\widetilde{u}_{\alpha L} - w'v\kappa'\pi'_\alpha\widetilde{u}'^*_L\widetilde{u}_{\alpha L} + H.c.)\big] \\
& + \frac{1}{18}\Big[v'^2\vartheta_i^2\widetilde{d}_{1L}\widetilde{d}^*_{1L} + u^2\Pi_{\alpha i}\Pi_{\beta i}\widetilde{d}_{\alpha L}\widetilde{d}^*_{\beta L} + w^2\Pi_{\alpha i}\Pi_{\beta i}\widetilde{d}'_{\alpha L}\widetilde{d}'^*_{\beta L} + \\
& + (uw\Pi_{\alpha i}\Pi_{\beta i}\widetilde{d}_{\alpha L}\widetilde{d}'^*_{\beta L} + v'u\vartheta_i\Pi_{\alpha i}\widetilde{d}_{1L}\widetilde{d}^*_{\alpha L} + v'w\vartheta_i\Pi_{\alpha i}\widetilde{d}_{1L}\widetilde{d}'^*_{\alpha L} + H.c.)\Big] \\
& + \frac{1}{18}\Big[v'^2\vartheta'^2_\delta\widetilde{d}_{1L}\widetilde{d}^*_{1L} + u^2\Pi'_{\alpha\delta}\Pi'_{\beta\delta}\widetilde{d}_{\alpha L}\widetilde{d}^*_{\beta L} + w^2\Pi'_{\alpha\delta}\Pi'_{\beta\delta}\widetilde{d}'_{\alpha L}\widetilde{d}'^*_{\beta L} + \\
& + (uw\Pi'_{\alpha\delta}\Pi'_{\beta\delta}\widetilde{d}_{\alpha L}\widetilde{d}'^*_{\beta L} + v'u\vartheta'_\delta\Pi'_{\alpha\delta}\widetilde{d}_{1L}\widetilde{d}^*_{\alpha L} + v'w\vartheta'_\delta\Pi'_{\alpha\delta}\widetilde{d}_{1L}\widetilde{d}'^*_{\alpha L} + H.c.)\Big] \\
= {} & D_Q + m^2_{u_{ij}}\widetilde{u}^{c*}_{iL}\widetilde{u}^c_{jL} + m^2_{d_{ij}}\widetilde{d}^{c*}_{iL}\widetilde{d}^c_{jL} + m^2_{u'}\widetilde{u'}^{c*}_L\widetilde{u'}^c_L + m^2_{d'\alpha\beta}\widetilde{d'}^{c*}_{\alpha L}\widetilde{d'}^c_{\beta L} \\
& + \Bigg[\frac{p_i}{\sqrt{2}}\left(u'\widetilde{u}_{1L} + w'\widetilde{u}'_L\right)\widetilde{u}^c_{iL} + \frac{p}{\sqrt{2}}\left(u'\widetilde{u}_{1L} + w'\widetilde{u}'_L\right)\widetilde{u'}^c_L
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{vp_{\alpha i}}{\sqrt{2}}\widetilde{u}_{\alpha L}\widetilde{u}^c_{iL}-\frac{vr_\alpha}{\sqrt{2}}\widetilde{u}_{\alpha L}\widetilde{u'}^c_L+\frac{v'h_i}{\sqrt{2}}\widetilde{d}_{1L}\widetilde{d}^c_{iL}\\
&+\frac{v'h'_\alpha}{\sqrt{2}}\widetilde{d}_{1L}\widetilde{d'}^c_{\alpha L}+\frac{h_{\alpha i}}{\sqrt{2}}(u\widetilde{d}_{\alpha L}+w\widetilde{d}'_{\alpha L})\widetilde{d}^c_{iL}+\frac{h'_{\alpha\beta}}{\sqrt{2}}\left(u\widetilde{d}_{\alpha L}+w\widetilde{d}'_{\alpha L}\right)\widetilde{d'}^c_{\beta L}\\
&+\frac{1}{6\sqrt{2}}\mu_\chi(\kappa_i(u\tilde{u}_{1L}+w\tilde{u}'_L)\tilde{u}^c_{iL}+\kappa'(u\tilde{u}_{1L}+w\tilde{u}'_L)\tilde{u}'^c_L)\\
&+\frac{1}{6\sqrt{2}}\mu_\chi\left(\Pi_{\alpha i}(u'\widetilde{d}_{\alpha L}+w'\widetilde{d}'_{\alpha L})\tilde{d}^c_{iL}+\Pi'_{\alpha\beta}(u'\widetilde{d}_{\alpha L}+w'\widetilde{d}'_{\alpha L})\tilde{d}'^c_{\beta L}\right)\\
&-\frac{1}{6\sqrt{2}}\mu_\rho\left(\pi_{\alpha i}v'\tilde{u}_{\alpha L}\tilde{u}^c_{iL}+\pi'_\alpha v'\tilde{u}_{\alpha L}\tilde{u}'^c_L\right)+\frac{v}{6\sqrt{2}}\mu_\rho\left(\vartheta_i\tilde{d}_{1L}\tilde{d}^c_{iL}+\vartheta'_\alpha\tilde{d}_{1L}\tilde{d}'^c_{\alpha L}\right)\\
&+\frac{1}{6\sqrt{2}}\mu_{0a}(\xi_{a\alpha j}(u'\tilde{d}_{\alpha L}+w'\tilde{d}'_{\alpha L})\tilde{d}^c_{jL}+\xi'_{a\alpha\beta}(u'\tilde{d}_{\alpha L}+w'\tilde{d}'_{\alpha L})\tilde{d}'^c_{\beta L})\\
&+\frac{1}{18}\lambda_a wv(\xi_{a\alpha i}\tilde{d}_{\alpha L}\tilde{d}^c_{iL}+\xi'_{a\alpha\beta}\tilde{d}_{\alpha L}\tilde{d}'^c_{\beta L})\\
&+\frac{1}{18}\lambda_a uv(\xi_{a\alpha i}\tilde{d}'_{\alpha L}\tilde{d}^c_{iL}+\xi'_{a\alpha\beta}\tilde{d}'_{\alpha L}\tilde{d}'^c_{\beta L})\\
&+\frac{1}{18}u'w'(\kappa_i^2+\kappa'^2)\widetilde{u}^*_{1L}\widetilde{u}'_L-\frac{1}{18}u'v(\kappa_i\pi_{\alpha i}+\kappa'\pi'_\alpha)\widetilde{u}^*_{1L}\widetilde{u}_{\alpha L}\\
&-\frac{1}{18}w'v(\kappa_i\pi_{\alpha i}+\kappa'\pi'_\alpha)\widetilde{u}'^*_L\widetilde{u}_{\alpha L}+\frac{1}{18}uw(\Pi_{\alpha i}\Pi_{\beta i}+\Pi'_{\alpha\delta}\Pi'_{\beta\delta})\widetilde{d}_{\alpha L}\widetilde{d}'^*_{\beta L}\\
&+\frac{1}{18}v'u(\vartheta_i\Pi_{\alpha i}+\vartheta'\Pi'_{\alpha\delta})\widetilde{d}_{1L}\widetilde{d}^*_{\alpha L}+\frac{1}{18}v'w(\vartheta_i\Pi_{\alpha i}+\vartheta'\Pi'_{\alpha\delta})\widetilde{d}_{1L}\widetilde{d}'^*_{\alpha L}+H.c.\Big]\\
&+\frac{1}{18}\left[(w'^2+u'^2)(\kappa_i\kappa_j\tilde{u}^{c*}_{iL}\tilde{u}^c_{jL}+\kappa'^2\tilde{u}'^{c*}_L\tilde{u}'^c_L+\kappa_i\kappa'\tilde{u}^{c*}_{iL}\tilde{u}'^c_L+\kappa_i\kappa'\tilde{u}^c_{iL}\tilde{u}'^{c*}_L)\right.\\
&\left.+v'^2(\vartheta_i\vartheta_j\tilde{d}^{c*}_{iL}\tilde{d}^c_{jL}+\vartheta'_\alpha\vartheta'_\beta\tilde{d}'^{c*}_{\alpha L}\tilde{d}'^c_{\beta L}+\vartheta_i\vartheta'_\alpha\tilde{d}^{c*}_{iL}\tilde{d}'^c_{\alpha L}+\vartheta_i\vartheta'_\alpha\tilde{d}^c_{iL}\tilde{d}'^{c*}_{\alpha L})\right]\\
&+\frac{1}{18}\left[v^2(\pi_{\alpha i}\pi_{\alpha j}\tilde{u}^{c*}_{iL}\tilde{u}^c_{jL}+\pi'^2_\alpha\tilde{u}'^{c*}_L\tilde{u}'^c_L+\pi_{\alpha i}\pi'_\alpha\tilde{u}^{c*}_{iL}\tilde{u}'^c_L+\pi_{\alpha i}\pi'_\alpha\tilde{u}^c_{iL}\tilde{u}'^{c*}_L)\right.\\
&+(w^2+u^2)(\Pi_{\alpha i}\Pi_{\alpha j}\tilde{d}^{c*}_{iL}\tilde{d}^c_{jL}+\Pi'_{\alpha\beta}\Pi'_{\alpha\delta}\tilde{d}'^{c*}_{\beta L}\tilde{d}'^c_{\delta L}\\
&\left.+\Pi_{\alpha i}\Pi'_{\alpha\beta}\tilde{d}^{c*}_{iL}\tilde{d}'^c_{\beta L}+\Pi_{\alpha i}\Pi'_{\alpha\beta}\tilde{d}^c_{iL}\tilde{d}'^{c*}_{\beta L})\right]\\
&+m^2_{Q1L}(\tilde{u}^*_{1L}\tilde{u}_{1L}+\tilde{d}^*_{1L}\tilde{d}_{1L}+\tilde{u}'^*_L\tilde{u}'_L)\\
&+m^2_{Q\alpha\beta L}(\tilde{u}^*_{\alpha L}\tilde{u}_{\beta L}+\tilde{d}^*_{\alpha L}\tilde{d}_{\beta L}+\tilde{d}'^*_{\alpha L}\tilde{d}'_{\beta L})\\
&+\frac{1}{18}\left[u'^2(\kappa_i^2+\kappa'^2)\widetilde{u}^*_{1L}\widetilde{u}_{1L}+w'^2(\kappa_i^2+\kappa'^2)\widetilde{u}'^*_L\widetilde{u}'_L+v^2(\pi_{\alpha i}\pi_{\beta i}+\pi'_\alpha\pi'_\beta)\widetilde{u}_{\alpha L}\widetilde{u}^*_{\beta L}\right]\\
&+\frac{1}{18}\left[v'^2(\vartheta_i^2+\vartheta'^2_\delta)\widetilde{d}_{1L}\widetilde{d}^*_{1L}+u^2(\Pi_{\alpha i}\Pi_{\beta i}+\Pi'_{\alpha\delta}\Pi'_{\beta\delta})\widetilde{d}_{\alpha L}\widetilde{d}^*_{\beta L}\right.\\
&\left.+w^2(\Pi_{\alpha i}\Pi_{\beta i}+\Pi'_{\alpha\delta}\Pi'_{\beta\delta})\widetilde{d}'_{\alpha L}\widetilde{d}'^*_{\beta L}\right]\\
=\;&D_Q+\left[m^2_{u_{ij}}+\frac{1}{18}(w'^2+u'^2)\kappa_i\kappa_j+\frac{1}{18}v^2\pi_{\alpha i}\pi_{\alpha j}\right]\widetilde{u}^{c*}_{iL}\widetilde{u}^c_{jL}\\
&+\left[m^2_{d_{ij}}+\frac{1}{18}v'^2\vartheta_i\vartheta_j+\frac{1}{18}(w^2+u^2)\Pi_{\alpha i}\Pi_{\alpha j}\right]\widetilde{d}^{c*}_{iL}\widetilde{d}^c_{jL}\\
&+\left[m^2_{u'}+\frac{1}{18}\kappa'^2(u'^2+w'^2)+\frac{1}{18}v^2\pi'^2_\alpha\right]\widetilde{u'}^{c*}_L\widetilde{u'}^c_L
\end{aligned}
$$

$$
\begin{aligned}
&+\left[m^2_{d'\alpha\beta}+\frac{1}{18}v'^2\vartheta'_\alpha\vartheta'_\beta+\frac{1}{18}(w^2+u^2)\Pi'_{\delta\beta}\Pi'_{\delta\alpha}\right]\widetilde{d'}^{c*}_{\alpha L}\widetilde{d'}^{c}_{\beta L}\\
&+[m^2_{Q1L}+\frac{1}{18}u'^2(\kappa_i^2+\kappa'^2)]\tilde{u}^*_{1L}\tilde{u}_{1L}\\
&+[m^2_{Q1L}+\frac{1}{18}v'^2(\vartheta_i^2+\vartheta'^2_\delta)]\tilde{d}^*_{1L}\tilde{d}_{1L}\\
&+[m^2_{Q1L}+\frac{1}{18}w'^2(\kappa_i^2+\kappa'^2)]\tilde{u}'^*_L\tilde{u}'_L\\
&+[m^2_{Q\alpha\beta L}+\frac{1}{18}v^2(\pi_{\alpha i}\pi_{\beta i}+\pi'_\alpha\pi'_\beta)]\tilde{u}^*_{\alpha L}\tilde{u}_{\beta L}\\
&+[m^2_{Q\alpha\beta L}+\frac{1}{18}u^2(\Pi_{\alpha i}\Pi_{\beta i}+\Pi'_{\alpha\delta}\Pi'_{\beta\delta})]\tilde{d}^*_{\alpha L}\tilde{d}_{\beta L}\\
&+[m^2_{Q\alpha\beta L}+\frac{1}{18}w^2(\Pi_{\alpha i}\Pi_{\beta i}+\Pi'_{\alpha\delta}\Pi'_{\beta\delta})]\tilde{d}'^*_{\alpha L}\tilde{d}'_{\beta L}\\
&+\Bigg\{\left(\frac{u'p_i}{\sqrt{2}}+\frac{u\mu_\chi\kappa_i}{6\sqrt{2}}\right)\widetilde{u}_{1L}\widetilde{u}^c_{iL}+\left(\frac{w'p_i}{\sqrt{2}}+\frac{w\mu_\chi\kappa_i}{6\sqrt{2}}\right)\widetilde{u}'_L\widetilde{u}^c_{iL}+\left(\frac{w'p}{\sqrt{2}}+\frac{w\mu_\chi\kappa'}{6\sqrt{2}}\right)\widetilde{u}'_L\widetilde{u}'^c_L\\
&+\left(\frac{u'p}{\sqrt{2}}+\frac{u\mu_\chi\kappa'}{6\sqrt{2}}\right)\widetilde{u}_{1L}\widetilde{u}'^c_L-\left(\frac{vp_{\alpha i}}{\sqrt{2}}+\frac{v'\mu_\rho\pi_{\alpha i}}{6\sqrt{2}}\right)\widetilde{u}_{\alpha L}\widetilde{u}^c_{iL}-\left(\frac{vr_\alpha}{\sqrt{2}}+\frac{v'\mu_\rho\pi'_\alpha}{6\sqrt{2}}\right)\widetilde{u}_{\alpha L}\widetilde{u}'^c_L\\
&+\left(\frac{v'h_i}{\sqrt{2}}+\frac{v\mu_\rho\vartheta_i}{6\sqrt{2}}\right)\widetilde{d}_{1L}\widetilde{d}^c_{iL}+\left(\frac{v'h'_\alpha}{\sqrt{2}}+\frac{v\mu_\rho\vartheta'_\alpha}{6\sqrt{2}}\right)\widetilde{d}_{1L}\widetilde{d}'^c_{\alpha L}\\
&+\left(\frac{uh_{\alpha i}}{\sqrt{2}}+\frac{u'\mu_\chi\Pi_{\alpha i}}{6\sqrt{2}}+\frac{u'\mu_{0a}\xi_{a\alpha i}}{6\sqrt{2}}+\frac{1}{18}\lambda_a wv\xi_{a\alpha i}\right)\widetilde{d}_{\alpha L}\widetilde{d}^c_{iL}\\
&+\left(\frac{wh_{\alpha i}}{\sqrt{2}}+\frac{w'\mu_\chi\Pi_{\alpha i}}{6\sqrt{2}}+\frac{w'\mu_{0a}\xi_{a\alpha i}}{6\sqrt{2}}+\frac{1}{18}\lambda_a uv\xi_{a\alpha i}\right)\widetilde{d}'_{\alpha L}\widetilde{d}^c_{iL}\\
&+\left(\frac{uh'_{\alpha\beta}}{\sqrt{2}}+\frac{u'\mu_\chi\Pi'_{\alpha\beta}}{6\sqrt{2}}+\frac{u'\mu_{0a}\xi'_{a\alpha\beta}}{6\sqrt{2}}+\frac{1}{18}\lambda_a wv\xi'_{a\alpha\beta}\right)\widetilde{d}_{\alpha L}\widetilde{d}'^c_{\beta L}\\
&+\left(\frac{wh'_{\alpha\beta}}{\sqrt{2}}+\frac{w'\mu_\chi\Pi'_{\alpha\beta}}{6\sqrt{2}}+\frac{w'\mu_{0a}\xi'_{a\alpha\beta}}{6\sqrt{2}}+\frac{1}{18}\lambda_a uv\xi'_{a\alpha\beta}\right)\widetilde{d}'_{\alpha L}\widetilde{d}'^c_{\beta L}\\
&+\frac{1}{18}\left[(w'^2+u'^2)\kappa_i\kappa'+v^2\pi_{\alpha i}\pi'_\alpha\right]\tilde{u}^c_{iL}\tilde{u}'^{c*}_L\\
&+\frac{1}{18}\left[v'^2\vartheta_i\vartheta'_\beta+(w^2+u^2)\Pi_{\alpha i}\Pi'_{\alpha\beta}\right]\tilde{d}^c_{iL}\tilde{d}'^{c*}_{\beta L}\\
&+\frac{1}{2}\widetilde{u}^*_{1L}\widetilde{u}'_L\left[\frac{1}{9}u'w'(\kappa_i^2+\kappa'^2)-\frac{g^2}{2}uw\frac{\cos 2\beta}{s^2_\beta}\right]-\frac{1}{18}u'v(\kappa_i\pi_{\alpha i}+\kappa'\pi'_\alpha)\widetilde{u}^*_{1L}\widetilde{u}_{\alpha L}\\
&-\frac{1}{18}w'v(\kappa_i\pi_{\alpha i}+\kappa'\pi'_\alpha)\widetilde{u}'^*_L\widetilde{u}_{\alpha L}+\frac{1}{18}uw(\Pi_{\alpha i}\Pi_{\beta i}+\Pi'_{\alpha\delta}\Pi'_{\beta\delta})\widetilde{d}_{\alpha L}\widetilde{d}'^*_{\beta L}\\
&+\frac{1}{18}v'u(\vartheta_i\Pi_{\alpha i}+\vartheta'\Pi'_{\alpha\delta})\widetilde{d}_{1L}\widetilde{d}^*_{\alpha L}+\frac{1}{18}v'w(\vartheta_i\Pi_{\alpha i}+\vartheta'\Pi'_{\alpha\delta})\widetilde{d}_{1L}\widetilde{d}'^*_{\alpha L}\\
&+\frac{g^2}{4}uw\frac{\cos 2\beta}{s^2_\beta}\widetilde{d}_{\alpha L}\widetilde{d}'^*_{\alpha L}+H.c.\Bigg\}
\end{aligned}
\tag{5.2}
$$

Looking at (5.2) we see that there is mixing among ordinary squarks with exotic squarks (with primes) and this produces $8\times 8$ matrix for up-squarks and $10\times 10$ matrix for down-squaks. Noting that exotic squarks carry lepton number $\pm 2$, we conclude that coefficients

of the mixture among ordinary and exotic squarks are very small. In the following, we will neglect such mixing.

### 5.2 The lepton number conservation limit

Remind that, in the SM, neutrinos are massless and lepton number is conserved. Let us consider the SM limit. The lepton-number conservation conditions imposed to (5.2) are the following:

$$w'p_i + \frac{w\mu_\chi\kappa_i}{6} = u'p + \frac{u\mu_\chi\kappa'}{6} = vr_\alpha + \frac{v'\mu_\rho\pi'_\alpha}{6} = v'h'_\alpha + \frac{v\mu_\rho\vartheta'_\alpha}{6} = 0, \tag{5.3}$$

$$wh_{\alpha i} + \frac{w'\mu_\chi\Pi_{\alpha i}}{6} + \frac{w'\mu_{0a}\xi_{a\alpha i}}{6} + \frac{1}{18}\lambda_a uv\xi_{a\alpha i} = 0, \tag{5.4}$$

$$uh'_{\alpha\beta} + \frac{u'\mu_\chi\Pi'_{\alpha\beta}}{6} + \frac{u'\mu_{0a}\xi'_{a\alpha\beta}}{6} + \frac{1}{18}\lambda_a wv\xi'_{a\alpha\beta} = 0, \tag{5.5}$$

$$(w'^2+u'^2)\kappa_i\kappa' + v^2\pi_{\alpha i}\pi'_\alpha = v'^2\vartheta_i\vartheta'_\beta + (w^2+u^2)\Pi_{\alpha i}\Pi'_{\alpha\beta} = 0, \tag{5.6}$$

$$\frac{1}{9}u'w'(\kappa_i^2+\kappa'^2) - \frac{g^2}{2}uw\frac{\cos 2\beta}{s_\beta^2} = w'v(\kappa_i\pi_{\alpha i} + \kappa'\pi'_\alpha) = 0, \tag{5.7}$$

$$\frac{1}{2}uw\left[\frac{1}{9}(\Pi_{\alpha i}\Pi_{\alpha i} + \Pi'_{\alpha\delta}\Pi'_{\alpha\delta}) + \frac{g^2}{2}\frac{\cos 2\beta}{s_\beta^2}\right] = v'w(\vartheta_i\Pi_{\alpha i} + \vartheta'\Pi'_{\alpha\delta}) = 0, \tag{5.8}$$

$$uw(\Pi_{2i}\Pi_{3i} + \Pi'_{2\delta}\Pi'_{3\delta}) = 0. \tag{5.9}$$

With imposition lepton-number conservation, the difficulties of large squark mixing, will be very much eased. Let us denote

$$\overline{A}_{ij} = m^2_{u_{ij}} + \frac{1}{18}(w'^2+u'^2)\kappa_i\kappa_j + \frac{1}{18}v^2\pi_{\alpha i}\pi_{\alpha j} - \frac{2}{3}g^2t^2H_1\delta_{ij}, \tag{5.10}$$

$$\overline{B}_{ij} = m^2_{d_{ij}} + \frac{1}{18}v'^2\vartheta_i\vartheta_j + \frac{1}{18}(w^2+u^2)\Pi_{\alpha i}\Pi_{\alpha j} + \frac{1}{3}g^2t^2H_1\delta_{ij}, \tag{5.11}$$

$$\overline{C} = m^2_{u'} + \frac{1}{18}\kappa'^2(u'^2+w'^2) + \frac{1}{18}v^2\pi'^2_\alpha - \frac{2}{3}g^2t^2H_1, \tag{5.12}$$

$$\overline{P}_{\alpha\beta} = m^2_{d'\alpha\beta} + \frac{1}{18}v'^2\vartheta'_\alpha\vartheta'_\beta + \frac{1}{18}(w^2+u^2)\Pi'_{\delta\beta}\Pi'_{\delta\alpha} + \frac{1}{3}g^2t^2H_1\delta_{\alpha\beta}, \tag{5.13}$$

$$\overline{E}_{u_{1L}} = m^2_{Q1L} + \frac{1}{18}u'^2(\kappa_i^2+\kappa'^2) + g^2\left(\frac{1}{2}H_3 + \frac{1}{2\sqrt{3}}H_8 + \frac{1}{3}t^2H_1\right), \tag{5.14}$$

$$\overline{E}_{d_{1L}} = m^2_{Q1L} + \frac{1}{18}v'^2(\vartheta_i^2+\vartheta'^2_\alpha) + g^2\left(-\frac{1}{2}H_3 + \frac{1}{2\sqrt{3}}H_8 + \frac{1}{3}t^2H_1\right), \tag{5.15}$$

$$\overline{E}_{u'_L} = m^2_{Q1L} + \frac{1}{18}w'^2(\kappa_i^2+\kappa'^2) - g^2\left(\frac{1}{\sqrt{3}}H_8 - \frac{1}{3}t^2H_1\right), \tag{5.16}$$

$$\overline{F}_{u_{\alpha\beta}} = m^2_{Q\alpha\beta L} + \frac{1}{18}v^2(\pi_{\alpha i}\pi_{\beta i} + \pi'_\alpha\pi'_\beta) + g^2\delta_{\alpha\beta}\left(\frac{1}{2}H_3 - \frac{1}{2\sqrt{3}}H_8\right), \tag{5.17}$$

$$\overline{F}_{d_{\alpha\beta}} = m^2_{Q\alpha\beta L} + \frac{1}{18}u^2(\Pi'_{\beta\delta}\Pi'_{\alpha\delta} + \Pi_{\alpha i}\Pi_{\beta i}) - g^2\delta_{\alpha\beta}\left(\frac{1}{2}H_3 + \frac{1}{2\sqrt{3}}H_8\right), \tag{5.18}$$

$$\overline{F}_{d'_{\alpha\beta}} = m^2_{Q\alpha\beta L} + \frac{1}{18}w^2(\Pi'_{\beta\delta}\Pi'_{\alpha\delta} + \Pi_{\alpha i}\Pi_{\beta i}) + g^2\delta_{\alpha\beta}\frac{1}{\sqrt{3}}H_8, \tag{5.19}$$

$$
\begin{aligned}
\overline{G}_{\alpha i} &= -\left(\frac{v p_{\alpha i}}{\sqrt{2}} + \frac{v' \mu_\rho \pi_{\alpha i}}{6\sqrt{2}}\right), && (5.20)\\
\overline{H}_i &= \frac{v' h_i}{\sqrt{2}} + \frac{v \mu_\rho \vartheta_i}{6\sqrt{2}}, && (5.21)\\
\overline{K}_{\alpha i} &= \frac{u h_{\alpha i}}{\sqrt{2}} + \frac{u' \mu_\chi \Pi_{\alpha i}}{6\sqrt{2}} + \frac{u' \mu_{0a} \xi_{a\alpha i}}{6\sqrt{2}} + \frac{1}{18}\lambda_a w v \xi_{a\alpha i}, && (5.22)\\
\overline{N}_{\alpha\beta} &= \frac{w h'_{\alpha\beta}}{\sqrt{2}} + \frac{w' \mu_\chi \Pi'_{\alpha\beta}}{6\sqrt{2}} + \frac{w' \mu_{0a} \xi'_{a\alpha\beta}}{6\sqrt{2}} + \frac{1}{18}\lambda_a u v \xi'_{a\alpha\beta} && (5.23)
\end{aligned}
$$

Then the mass Lagrangian (5.2) can be rewritten in the form

$$
\begin{aligned}
\mathcal{L}_{squarks} &= \overline{C} \tilde{u}'^{c*}_L \tilde{u}'^{c}_L + \overline{E}_{u'_L} \tilde{u}'^{*}_L \tilde{u}'_L + \overline{E}_{u_{1L}} \tilde{u}^*_{1L} \tilde{u}_{1L} + \overline{E}_{d_{1L}} \tilde{d}^*_{1L} \tilde{d}_{1L} \\
&+ \overline{A}_{ij} \tilde{u}^{c*}_{iL} \tilde{u}^c_{jL} + \overline{P}_{\alpha\beta} \tilde{d}'^{c*}_{\alpha L} \tilde{d}'^{c}_{\beta L} + \overline{F}_{u_{\alpha\beta}} \tilde{u}^*_{\alpha L} \tilde{u}_{\beta L} \\
&+ \overline{B}_{ij} \tilde{d}^{c*}_{iL} \tilde{d}^c_{jL} + \overline{F}_{d'_{\alpha\beta}} \tilde{d}'^{*}_{\alpha L} \tilde{d}'_{\beta L} + \overline{F}_{d_{\alpha\beta}} \tilde{d}^*_{\alpha L} \tilde{d}_{\beta L} \\
&+ [\overline{K}_{\alpha i} \tilde{d}_{\alpha L} \tilde{d}^c_{iL} + \overline{N}_{\alpha\beta} \tilde{d}'_{\alpha L} \tilde{d}'^{c}_{\beta L} + \overline{G}_{\alpha i} \tilde{u}_{\alpha L} \tilde{u}^c_{iL} + \overline{H}_i \tilde{d}_{1L} \tilde{d}^c_{iL} + H.c.] \qquad (5.24)
\end{aligned}
$$

From (5.24) we see that all the exotic squarks are decoupled of ordinary squarks. The reason of this is that in the 3-3-1 models, the exotic quarks carry also lepton number, while the ordinary ones do not, so their superpartners have the same property.

Looking at (5.24), we conclude that the $\tilde{u}'^{c}_L$ and $\tilde{u}'_L$ gain masses respectively,

$$
m^2_{\tilde{u}'^{c}_L} = \overline{C}, \qquad m^2_{\tilde{u}'_L} = \overline{E}_{u'_L} \qquad (5.25)
$$

For the ordinary up-squarks, the $\tilde{u}_{1L}$ does not mix and gains mass:

$$
m^2_{\tilde{u}_{1L}} = \overline{E}_{u_{1L}}. \qquad (5.26)
$$

The remaining up-squarks are all mixing and in the base $(\tilde{u}^*_{2L}, \tilde{u}^*_{3L}, \tilde{u}^c_{1L}, \tilde{u}^c_{2L}, \tilde{u}^c_{3L})$, the mass matrix is given by

$$
\begin{pmatrix}
\overline{F}_{u_{22}} & \overline{F}_{u_{23}} & \overline{G}_{21} & \overline{G}_{22} & \overline{G}_{23} \\
\overline{F}_{u_{32}} & \overline{F}_{u_{33}} & \overline{G}_{31} & \overline{G}_{32} & \overline{G}_{33} \\
\overline{G}_{21} & \overline{G}_{31} & \overline{A}_{11} & \overline{A}_{21} & \overline{A}_{31} \\
\overline{G}_{22} & \overline{G}_{32} & \overline{A}_{12} & \overline{A}_{22} & \overline{A}_{32} \\
\overline{G}_{23} & \overline{G}_{33} & \overline{A}_{13} & \overline{A}_{23} & \overline{A}_{33}
\end{pmatrix}. \qquad (5.27)
$$

For superpartners of the down-quarks ($q = -\frac{1}{3}$), we have: The $\tilde{d}'^{*}_{2L}$, $\tilde{d}'^{*}_{3L}$, $\tilde{d}'^{c}_{2L}$ and $\tilde{d}'^{c}_{3L}$ mix with mass matrix

$$
\begin{pmatrix}
\overline{F}_{d'_{22}} & \overline{F}_{d'_{23}} & \overline{N}_{22} & \overline{N}_{23} \\
\overline{F}_{d'_{32}} & \overline{F}_{d'_{33}} & \overline{N}_{32} & \overline{N}_{33} \\
\overline{N}_{22} & \overline{N}_{32} & \overline{P}_{22} & \overline{P}_{32} \\
\overline{N}_{23} & \overline{N}_{33} & \overline{P}_{23} & \overline{P}_{33}
\end{pmatrix}. \qquad (5.28)
$$

For the ordinary down-squark,in the base $(\tilde{d}^*_{1L}, \tilde{d}^*_{2L}, \tilde{d}^*_{3L}, \tilde{d}^c_{1L}, \tilde{d}^c_{2L}, \tilde{d}^c_{3L})$, the mass matrix is defined by

$$\begin{pmatrix} \overline{E}_{d_{1L}} & 0 & 0 & \overline{H}_1 & \overline{H}_2 & \overline{H}_3 \\ 0 & \overline{F}_{d_{22}} & \overline{F}_{d_{23}} & \overline{K}_{21} & \overline{K}_{22} & \overline{K}_{23} \\ 0 & \overline{F}_{d_{32}} & \overline{F}_{d_{33}} & \overline{K}_{31} & \overline{K}_{32} & \overline{K}_{33} \\ \overline{H}_1 & \overline{K}_{21} & \overline{K}_{31} & \overline{B}_{11} & \overline{B}_{21} & \overline{B}_{31} \\ \overline{H}_2 & \overline{K}_{22} & \overline{K}_{32} & \overline{B}_{12} & \overline{B}_{22} & \overline{B}_{32} \\ \overline{H}_3 & \overline{K}_{23} & \overline{K}_{33} & \overline{B}_{13} & \overline{B}_{23} & \overline{B}_{33} \end{pmatrix}. \tag{5.29}$$

As in the MSSM, the ordinary down-squarks mix by $6 \times 6$ matrix.

It is to be noted that the decoupling of $\tilde{u}_{1L}$ is a result of the condition for minimum of Higgs potential: $u'/u = w'/w$ and the absence of $Q_{1L}$ in two last terms of Eq. (2.19).

In general, we cannot deal with $5\times5$ and $6\times6$ matrices. Following Refs [14] and [23], we expect the $\tilde{q}_L - \tilde{q}_R$ mixing to be small, with possible exception of the third-generation, where mixing can be enhanced by factors of $m_t$ and $m_b$. Keeping in mind this assumption, from Eq. (5.27) to Eq. (5.29), we conclude that, all non-vanishing matrix elements are: $\overline{F}_{u_{22}}$, $\overline{F}_{u_{33}}$, $\overline{A}_{11}$, $\overline{A}_{22}$, $\overline{A}_{33}$, $\overline{G}_{33}$, $\overline{F}_{d'_{22}}$, $\overline{F}_{d'_{33}}$,$\overline{P}_{22}$, $\overline{P}_{33}$,$\overline{N}_{33}$, $\overline{E}_{d_{1L}}$,$\overline{F}_{d_{22}}$, $\overline{F}_{d_{33}}$, $\overline{B}_{11}$, $\overline{B}_{22}$, $\overline{B}_{33}$ and $\overline{K}_{33}$ With the help of the above assumption, diagonalization of the mass mixing matrices is quite easy. Our results are as follows:

1. *For up-squarks*:

   The eigenmasses and eigenstates are given in Table 3. and two others are

**Table 3:** Masses and eigenstates of up-squarks

| Eigenstate | $\tilde{u}_{1L}$ | $\tilde{u}_{1R}$ | $\tilde{u}_{2L}$ | $\tilde{u}_{2R}$ | $\tilde{u}'_L$ | $\tilde{u}'_R$ |
|---|---|---|---|---|---|---|
| ( Mass)$^2$ | $\overline{E}_{u_{1L}}$ | $\overline{A}_{11}$ | $\overline{F}_{u_{22}}$ | $\overline{A}_{22}$ | $\overline{E}_{u'_L}$ | $\overline{C}$ |

$$\tilde{u}_{t_L} = s_{\theta_u}\tilde{u}_{3R} - c_{\theta_u}\tilde{u}_{3L}, \tag{5.30}$$
$$\tilde{u}_{t_R} = c_{\theta_u}\tilde{u}_{3R} + s_{\theta_u}\tilde{u}_{3L}, \tag{5.31}$$

with respective masses

$$m^2_{\tilde{u}_{tL}} = \frac{1}{2}(\overline{F}_{u_{33}} + \overline{A}_{33} - \overline{\Delta}), \tag{5.32}$$
$$m^2_{\tilde{u}_{tR}} = \frac{1}{2}(\overline{F}_{u_{33}} + \overline{A}_{33} + \overline{\Delta}), \tag{5.33}$$

where

$$\overline{\Delta} = \sqrt{(\overline{A}_{33} - \overline{F}_{u_{33}})^2 + 4\overline{G}^2_{33}}, \tag{5.34}$$
$$t_{2\theta_u} = \frac{2\overline{G}_{33}}{\overline{A}_{33} - \overline{F}_{u_{33}}} \tag{5.35}$$

**Table 4:** Masses and eigenstates of down-squarks

| Eigenstate | $\tilde{d}_{1L}$ | $\tilde{d}_{1R}$ | $\tilde{d}_{2L}$ | $\tilde{d}_{2R}$ | $\tilde{d}'_{2L}$ | $\tilde{d}'_{2R}$ |
|---|---|---|---|---|---|---|
| ( Mass)$^2$ | $\overline{E}_{d_{1L}}$ | $\overline{B}_{11}$ | $\overline{F}_{d_{22}}$ | $\overline{B}_{22}$ | $\overline{F}_{d'_{22}}$ | $\overline{P}_{22}$ |

2. *For down-squarks*:

   Eigenstates and masses are presented in Table 4.

and four others are

$$\tilde{d}_{t_L} = s_{\theta_d}\tilde{d}_{3R} - c_{\theta_d}\tilde{d}_{3L}, \tag{5.36}$$
$$\tilde{d}_{t_R} = c_{\theta_d}\tilde{d}_{3R} + s_{\theta_d}\tilde{d}_{3L}, \tag{5.37}$$

with respective masses

$$m^2_{\tilde{d}_{tL}} = \frac{1}{2}(\overline{F}_{d_{33}} + \overline{B}_{33} - \overline{\Delta}), \tag{5.38}$$
$$m^2_{\tilde{d}_{tR}} = \frac{1}{2}(\overline{F}_{d_{33}} + \overline{B}_{33} + \overline{\Delta}), \tag{5.39}$$

where

$$\overline{\Delta} = \sqrt{(\overline{B}_{33} - \overline{F}_{d_{33}})^2 + 4\overline{K}^2_{33}}, \tag{5.40}$$
$$t_{2\theta_d} = \frac{2\overline{K}_{33}}{\overline{B}_{33} - \overline{F}_{d_{33}}}. \tag{5.41}$$

Analogously for exotic squarks:

$$\tilde{d}'_{t_L} = s_{\theta_{d'}}\tilde{d}'_{3R} - c_{\theta_{d'}}\tilde{d}'_{3L}, \tag{5.42}$$
$$\tilde{d}'_{t_R} = c_{\theta_{d'}}\tilde{d}'_{3R} + s_{\theta_{d'}}\tilde{d}'_{3L}, \tag{5.43}$$

with respective masses

$$m^2_{\tilde{d}'_{tL}} = \frac{1}{2}(\overline{F}_{d'_{33}} + \overline{P}_{33} - \overline{\Delta}), \tag{5.44}$$
$$m^2_{\tilde{d}'_{tR}} = \frac{1}{2}(\overline{F}_{d'_{33}} + \overline{P}_{33} + \overline{\Delta}), \tag{5.45}$$

where

$$\overline{\Delta} = \sqrt{(\overline{P}_{33} - \overline{F}_{d'_{33}})^2 + 4\overline{N}^2_{33}}, \tag{5.46}$$
$$t_{2\theta_{d'}} = \frac{2\overline{N}_{33}}{\overline{P}_{33} - \overline{F}_{d'_{33}}}. \tag{5.47}$$

To outline mass spectrum, let us assume

$$\begin{aligned} &\overline{A}_{11} < \overline{C} < \overline{B}_{11} < \overline{E}_{u'_L} < \overline{E}_{u_{1L}} < \overline{E}_{d_{1L}}, \\ &\overline{A}_{22} < \overline{B}_{22} < \overline{P}_{22} < \overline{F}_{u_{22}} < \overline{F}_{d_{22}} < \overline{F}_{d'_{22}}. \end{aligned} \tag{5.48}$$

Squarks mass spectrum is shown in figure 1, where mass scales between generations are not taken into account.

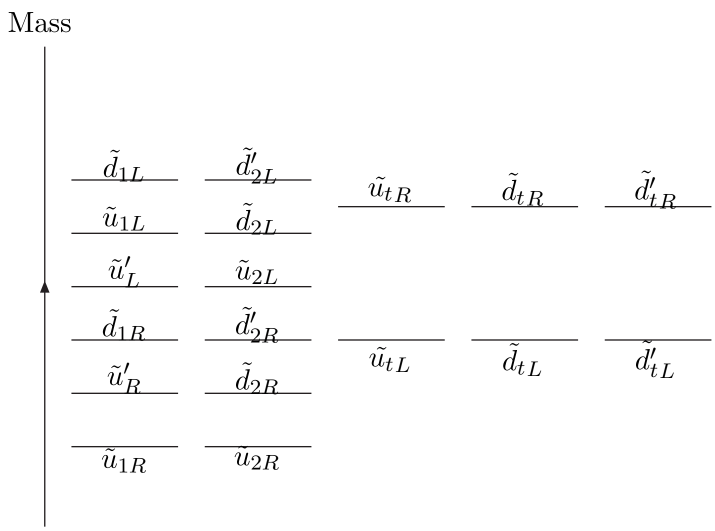


**Figure 1:** A schematic sample mass spectrum for squarks in which mass scales between generations are not taken into account.

We summarize this section by notice that the huge squark mixing matrices ($8 \times 8$ and $10 \times 10$) were significally reduced by the lepton number conservation. The situation will be much better by $R$-parity imposition.

## 6. $R$-parity and sfermion mass splitting

Consequence of $R$-parity is that all coefficients in $W_{\not{R}}$ and $\mathcal{L}_{SMT}^{\not{R}}$ vanish.

### 6.1 Slepton mass splitting

$R$-parity conservation and the constraint (2.20) give

$$\lambda_a = \lambda'_{ab} = M'^2_a = \upsilon_a = \mu_{0a} = 0. \tag{6.1}$$

Then vanishing of nondiagonal elements in lepton mixing matrices leads to:

$$\gamma_{c1}\gamma_{c2} = \gamma_{c1}\gamma_{c3} = \gamma_{c3}\gamma_{c2} = 0. \tag{6.2}$$

Consequence of (6.2) is that at least, one of the coefficients $\gamma_{ab}$ vanishes. Let us consider two special cases:

1. $\gamma_{c3} \neq 0$

   From (6.2) we get

$$\gamma_{c1} = \gamma_{c2} = 0, \quad \Rightarrow \quad \gamma_{33} \neq 0. \tag{6.3}$$

In the considering case, the first two family slepton masses are given by:

$$m^2_{\tilde{l}_{1L}} = M^2_{11} - \frac{g^2}{2}\left(H_3 - \frac{1}{\sqrt{3}}H_8 + \frac{2t^2}{3}H_1\right), \tag{6.4}$$

$$m^2_{\tilde{\nu}_{1L}} = M^2_{11} + \frac{g^2}{2}\left(H_3 + \frac{1}{\sqrt{3}}H_8 - \frac{2t^2}{3}H_1\right), \tag{6.5}$$

$$m^2_{\tilde{\nu}_{1R}} = M^2_{11} - g^2\left(\frac{1}{\sqrt{3}}H_8 + \frac{t^2}{3}H_1\right), \tag{6.6}$$

$$m^2_{\tilde{l}_{1R}} = m^2_{11} + g^2t^2H_1, \tag{6.7}$$

$$m^2_{\tilde{l}_{2L}} = M^2_{22} - \frac{g^2}{2}\left(H_3 - \frac{1}{\sqrt{3}}H_8 + \frac{2t^2}{3}H_1\right), \tag{6.8}$$

$$m^2_{\tilde{\nu}_{2L}} = M^2_{22} + \frac{g^2}{2}\left(H_3 + \frac{1}{\sqrt{3}}H_8 - \frac{2t^2}{3}H_1\right), \tag{6.9}$$

$$m^2_{\tilde{\nu}_{2R}} = M^2_{22} - g^2\left(\frac{1}{\sqrt{3}}H_8 + \frac{t^2}{3}H_1\right), \tag{6.10}$$

$$m^2_{\tilde{l}_{2R}} = m^2_{22} + g^2t^2H_1, \tag{6.11}$$

The stau masses are defined:

$$m^2_{\tilde{\tau}_L} = \frac{1}{2}\left[M^2_{33} + m^2_{33} + \frac{v'^2}{9}\gamma^2_{33} - \frac{g^2}{2}\left(H_3 - \frac{1}{\sqrt{3}}H_8 - \frac{2t^2}{3}H_1\right) - \Delta_1\right], \tag{6.12}$$

$$m^2_{\tilde{\tau}_R} = \frac{1}{2}\left[M^2_{33} + m^2_{33} + \frac{v'^2}{9}\gamma^2_{33} - \frac{g^2}{2}\left(H_3 - \frac{1}{\sqrt{3}}H_8 - \frac{2t^2}{3}H_1\right) + \Delta_1\right], \tag{6.13}$$

where

$$\Delta_1 = \sqrt{\left[M^2_{33} - m^2_{33} - g^2\left(H_3 - \frac{1}{\sqrt{3}}H_8 + \frac{8t^2}{3}H_1\right)\right]^2 + 2\left(\eta_{33}v' + \frac{1}{6}\mu_\rho\gamma_{33}v\right)^2}. \tag{6.14}$$

For the mixing sneutrino eigenstates

$$\tilde{\nu}_{\tau L} = s_{\theta_n}\tilde{\nu}_{3R} - c_{\theta_n}\tilde{\nu}_{3L}, \tag{6.15}$$

$$\tilde{\nu}_{\tau R} = c_{\theta_n}\tilde{\nu}_{3R} + s_{\theta_n}\tilde{\nu}_{3L}, \tag{6.16}$$

we obtain

$$m^2_{\tilde{\nu}_{\tau L}} = \frac{1}{2}\left[2M^2_{33} + \frac{g^2}{2}\left(H_3 - \frac{1}{\sqrt{3}}H_8 - \frac{4t^2}{3}H_1\right) - \Delta_{n1}\right], \tag{6.17}$$

$$m^2_{\tilde{\nu}_{\tau R}} = \frac{1}{2}\left[2M^2_{33} + \frac{g^2}{2}\left(H_3 - \frac{1}{\sqrt{3}}H_8 - \frac{4t^2}{3}H_1\right) + \Delta_{n1}\right], \tag{6.18}$$

with

$$\Delta_{n1} = \sqrt{\frac{g^4}{4}\left(H_3 + \sqrt{3}H_8\right)^2 + 8\left(\varepsilon_{33}v + \frac{1}{6}\mu_\rho\lambda'_{33}v'\right)^2}. \tag{6.19}$$

From Eq. (6.4) to Eq. (6.11), the mass splittings for the sleptons are governed by sum-rules

$$m^2_{\tilde{l}_{1L}} - m^2_{\tilde{\nu}_{1L}} = m^2_{\tilde{l}_{2L}} - m^2_{\tilde{\nu}_{2L}} = -g^2 H_3 = \frac{g^2}{4}\left(v^2\frac{\cos 2\gamma}{c^2_\gamma} + u^2\frac{\cos 2\beta}{s^2_\beta}\right)$$
$$= m^2_W \cos 2\gamma + \frac{g^2u^2}{4}\frac{\cos 2\beta}{s^2_\beta}, \quad (6.20)$$
$$m^2_{\tilde{\nu}_{1L}} - m^2_{\tilde{\nu}_{1R}} = m^2_{\tilde{\nu}_{2L}} - m^2_{\tilde{\nu}_{2R}} = \frac{g^2}{2}\left(H_3 + \sqrt{3}H_8\right) = \frac{g^2}{4}(w^2 - u^2)\frac{\cos 2\beta}{s^2_\beta} \quad (6.21)$$

Remind that, in the effective approximation, we have [18, 19]: $w \simeq w', u \simeq u'$. Thus, noting that our notation $\tan\gamma$ is $\cot\beta$ in MSSM as in Ref. [14], Eq. (6.20) coincides with the MSSM result [14]. In this approximation, there is degeneration among $\tilde{\nu}_{1(2)L}$ and $\tilde{\nu}_{1(2)R}$. As in the MSSM, $\cos 2\gamma > 0$ in the allowed range $\tan\gamma < 1$, we get then $m^2_{\tilde{l}_L} > m^2_{\tilde{\nu}_{lL}}$, $l = e, \mu$. Assuming further $\cos 2\beta > 0$, we obtain: $m^2_{\tilde{\nu}_{lL}} > m^2_{\tilde{\nu}_{lR}}$.
To outline slepton mass spectrum, we assume the following relationship:

$$\cos 2\gamma > 0, \quad \cos 2\beta > 0, \quad m^2_{11} < m^2_{22}. \quad (6.22)$$

With the above assumption, the slepton mass spectrum is shown in figure 2. Since, no convincing evidence for production of superpartners has been found, our figure has only illustrative meaning.

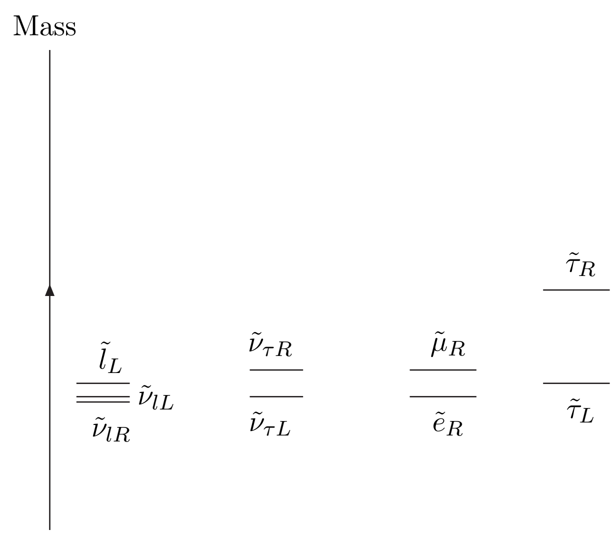


**Figure 2:** A schematic sample mass spectrum for sleptons, in which mass scales between generations are not taken into account and $l = e, \mu$.

2. $\gamma_{c1} \neq 0$

As before, from (6.2) we have

$$\gamma_{c2} = \gamma_{c3} = 0, \quad \Rightarrow \quad \gamma_{11} \neq 0. \tag{6.23}$$

In this case, all the charged sleptons have different masses:

$$m^2_{\tilde{l}_{1L}} = M^2_{11} + \frac{v'^2}{18}\gamma^2_{11} - \frac{g^2}{2}\left(H_3 - \frac{1}{\sqrt{3}}H_8 + \frac{2t^2}{3}H_1\right), \tag{6.24}$$

$$m^2_{\tilde{l}_{2L}} = M^2_{22} - \frac{g^2}{2}\left(H_3 - \frac{1}{\sqrt{3}}H_8 + \frac{2t^2}{3}H_1\right), \tag{6.25}$$

$$m^2_{\tilde{l}_{1R}} = m^2_{11} + \frac{v'^2}{18}\gamma^2_{11} + g^2t^2H_1, \tag{6.26}$$

$$m^2_{\tilde{l}_{2R}} = m^2_{22} + g^2t^2H_1. \tag{6.27}$$

and

$$m^2_{\tilde{\tau}_L} = \frac{1}{2}\left[M^2_{33} + m^2_{33} - \frac{g^2}{2}\left(H_3 - \frac{1}{\sqrt{3}}H_8 - \frac{2t^2}{3}H_1\right) - \Delta_2\right], \tag{6.28}$$

$$m^2_{\tilde{\tau}_R} = \frac{1}{2}\left[M^2_{33} + m^2_{33} - \frac{g^2}{2}\left(H_3 - \frac{1}{\sqrt{3}}H_8 - \frac{2t^2}{3}H_1\right) + \Delta_2\right] \tag{6.29}$$

with

$$\Delta_2 = \sqrt{\left[M^2_{33} - m^2_{33} - \frac{g^2}{2}\left(H_3 - \frac{1}{\sqrt{3}}H_8 + \frac{8t^2}{3}H_1\right)\right]^2 + 2\eta^2_{33}v'^2}. \tag{6.30}$$

For sneutrinos, we have

$$m^2_{\tilde{\nu}_{1L}} = M^2_{11} + \frac{g^2}{2}\left(H_3 + \frac{1}{\sqrt{3}}H_8 - \frac{2t^2}{3}H_1\right), \tag{6.31}$$

$$m^2_{\tilde{\nu}_{1R}} = M^2_{11} - g^2\left(\frac{1}{\sqrt{3}}H_8 + \frac{t^2}{3}H_1\right), \tag{6.32}$$

$$m^2_{\tilde{\nu}_{2L}} = M^2_{22} + \frac{g^2}{2}\left(H_3 + \frac{1}{\sqrt{3}}H_8 - \frac{2t^2}{3}H_1\right), \tag{6.33}$$

$$m^2_{\tilde{\nu}_{2R}} = M^2_{22} - g^2\left(\frac{1}{\sqrt{3}}H_8 + \frac{t^2}{3}H_1\right). \tag{6.34}$$

For the tau sneutrinos, we get

$$m^2_{\tilde{\nu}_{\tau L}} = \frac{1}{2}\left[2M^2_{33} + \frac{g^2}{2}\left(H_3 - \frac{1}{\sqrt{3}}H_8 - \frac{4t^2}{3}H_1\right) - \Delta_n\right], \tag{6.35}$$

$$m^2_{\tilde{\nu}_{\tau R}} = \frac{1}{2}\left[2M^2_{33} + \frac{g^2}{2}\left(H_3 - \frac{1}{\sqrt{3}}H_8 - \frac{4t^2}{3}H_1\right) + \Delta_n\right], \tag{6.36}$$

with

$$\Delta_n = \sqrt{\frac{g^4}{4}\left(H_3 + \sqrt{3}H_8\right)^2 + 8\left(\varepsilon_{33}v + \frac{1}{6}\mu_\rho\lambda'_{33}v'\right)^2}. \tag{6.37}$$

In the considered case, only relation among sneutrinos (6.21) is satisfied.

In this case, slepton mass spectrum is similar to figure 2.

The situation is similar for case of $\gamma_{c2} \neq 0$, in which the second generation plays a role of the first one. Next, let us consider squarks in the model under consideration.

### 6.2 Squark mass mixing matrices

Imposition of $R-parity$ yields

$$\xi_{a\alpha i} = \xi'_{a\alpha\beta} = \lambda_a = 0. \tag{6.38}$$

Looking at (5.4)–(5.5) we have

$$\overline{K}_{\alpha i} = \overline{N}_{\alpha\beta} = 0. \tag{6.39}$$

So that the mass matrix of down-squarks is significally reduced.

With the help of (6.39), matrix in (5.28) is decomposed to two $2\times 2$ ones. $\tilde{d}'^{c}_{2L}$ and $\tilde{d}'^{c}_{3L}$ mix with mass matrix

$$\begin{pmatrix} \overline{P}_{22} & \overline{P}_{23} \\ \overline{P}_{32} & \overline{P}_{33} \end{pmatrix}. \tag{6.40}$$

For $\tilde{d}'_{2L}$ and $\tilde{d}'_{3L}$, the mass matrix is

$$\begin{pmatrix} \overline{F}_{d'_{22}} & \overline{F}_{d'_{23}} \\ \overline{F}_{d'_{32}} & \overline{F}_{d'_{33}} \end{pmatrix}. \tag{6.41}$$

For the ordinary down-squark, matrix in (5.29) is decomposed into $2\times 2$ and $4\times 4$ ones. We have two blocks: in the base $(\tilde{d}_{2L}, \tilde{d}_{3L})$, the mass matrix is given by

$$\begin{pmatrix} \overline{F}_{d_{22}} & \overline{F}_{d_{23}} \\ \overline{F}_{d_{32}} & \overline{F}_{d_{33}} \end{pmatrix}. \tag{6.42}$$

Four others mix and, in the base $(\tilde{d}^*_{1L}, \tilde{d}^c_{1L}, \tilde{d}^c_{2L}, \tilde{d}^c_{3L})$, the mass matrix is defined by

$$\begin{pmatrix} \overline{E}_{d_{1L}} & \overline{H}_1 & \overline{H}_2 & \overline{H}_3 \\ \overline{H}_1 & \overline{B}_{11} & \overline{B}_{21} & \overline{B}_{31} \\ \overline{H}_2 & \overline{B}_{12} & \overline{B}_{22} & \overline{B}_{32} \\ \overline{H}_3 & \overline{B}_{13} & \overline{B}_{23} & \overline{B}_{33} \end{pmatrix}. \tag{6.43}$$

It is interesting to note that our highest mass mixing matrices are smaller than that in the MSSM ($6\times 6$ matrices).

Let us consider the squark mass splitting. Looking at Eqs.(5.14)-(5.19) yields the mass splitting of squarks in the first generation:

$$\begin{aligned} m^2_{\tilde{d}_{1L}} - m^2_{\tilde{u}_{1L}} &= \overline{E}_{d_{1L}} - \overline{E}_{u_{1L}} = -g^2 H_3 + \frac{1}{18}\left[v'^2(\vartheta_i^2 + \vartheta'^2_\alpha) - u'^2(\kappa_i^2 + \kappa'^2)\right] \\ &= \frac{g^2}{4}\left(v^2 \frac{\cos 2\gamma}{c^2_\gamma} + u^2 \frac{\cos 2\beta}{s^2_\beta}\right) + \frac{1}{18}\left[v'^2(\vartheta_i^2 + \vartheta'^2_\alpha) - u'^2(\kappa_i^2 + \kappa'^2)\right] \\ &= m^2_W \cos 2\gamma + \frac{g^2 u^2}{4}\frac{\cos 2\beta}{s^2_\beta} + \frac{1}{18}\left[v'^2(\vartheta_i^2 + \vartheta'^2_\alpha) - u'^2(\kappa_i^2 + \kappa'^2)\right] \\ &\simeq m^2_W \cos 2\gamma + \frac{1}{18} v'^2 \vartheta_i^2. \end{aligned} \tag{6.44}$$

(6.45)

Similarly, for the second generation, we get

$$\begin{aligned}
m^2_{\tilde{d}_{2L}} - m^2_{\tilde{u}_{2L}} &= \overline{F}_{d_{22}} - \overline{F}_{u_{22}} = -g^2 H_3 \\
&+ \frac{1}{18}[u^2(\Pi'_{\beta\delta}\Pi'_{\alpha\delta} + \Pi_{\beta i}\Pi_{\alpha i}) - v^2(\pi_{\beta i}\pi_{\alpha i} + \pi'_{\beta}\pi'_{\alpha})] \\
&\simeq m_W^2 \cos 2\gamma - \frac{1}{18} v^2 \pi_{\beta i}\pi_{\alpha i}. \qquad (6.46)
\end{aligned}$$

In the model under consideration, squark mass splitting is different from those in the MSSM and the reason of this is the quark generation discrimination. For sleptons, the splitting is the same as in the MSSM. In addition, in the SM limit, we have triple degeneracy among all particles in the lepton triplet.

## 7. Conclusions

In this paper we have studied the sfermion sector in the supersymmetric economical 3-3-1 model. Our calculation of the full superpotential for sfermions is useful for further study on searching of supersymmetric particles at high energy colliders such as the CERN Large Hadron Collider (LHC),... By $R$- parity conservation imposition, the Higgs scalars are decoupled of the sfermions; and the exotic squarks are also decoupled of superpartners of the ordinary quarks.

In contradiction to the MSSM, in the model under consideration, there are lepton number violating mass terms in the contribution from $D$-part.

As in the MSSM, the mass mixing matrix for charged sleptons is $6 \times 6$, while for sneutrinos, due to the existence of the right-handed neutrinos, their mass mixing matrix is $6 \times 6$ too (Remind that in the MSSM, it is $3 \times 3$ matrix).

It is worth noting that, in the SM limit, due to the quark family discrimination, the highest mass mixing matrices for the up-squarks and the down-squarks are, respectively, $5 \times 5$ and $4 \times 4$, but not $6 \times 6$ as in the MSSM. Due to the same reason, in contradiction with the MSSM, there is no mixing among $\tilde{b}_L$ and $\tilde{b}_R$.

Assuming that there is only mixing among highest flavors ( $\tilde{\nu}_{\tau L} - \tilde{\nu}_{\tau R}$, $\tilde{\tau}_L - \tilde{\tau}_R$ and $\tilde{t}_L - \tilde{t}_R$) we were able to outline mass spectra for the sfermions in the model.

In the SM limit, without $D$-term contribution, there is triple degeneracy among all particles in the lepton triplet. Therefore the mass splitting among sleptons is proportional the $D$-term contribution

$$\begin{aligned}
m^2_{\tilde{l}_L} - m^2_{\tilde{\nu}_L} &\simeq m_W^2 \cos 2\gamma, \\
m^2_{\tilde{\nu}_{eL}} &= m^2_{\tilde{\nu}_{eR}}. \qquad (7.1)
\end{aligned}$$

However, due to the quark generation discrimination, squark mass splittings are different in each family and from those in the MSSM.

We do hope that, in coming years, the CERN LHC will provide important information on the supersymmetric particles including sfermions and our prediction in Eq. (7.1) can be experimentally checked.

To conclude this work, we note again that due to the minimal content of the scalar sector, in the supersymmetric economical 3-3-1 model, Higgs sector is quite constrained and the significant number of free parameters is reduced. Its supersymmetric extension has the same feature and deserves further studies.

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## References

[1] F. Pisano and V. Pleitez, Phys. Rev. D **46** (1992) 410; P. H. Frampton, Phys. Rev. Lett. **69** (1992) 2889; R. Foot *et al.*, Phys. Rev. D **47** (1993) 4158.

[2] M. Singer *et al.*, Phys. Rev. D **22** (1980) 738.

[3] R. Foot, H. N. Long and Tuan A. Tran, Phys. Rev. D **50** (1994) 34; J. C. Montero *et al.*, Phys. Rev. D **47** (1993) 2918; H. N. Long, Phys. Rev. D **54** (1996) 4691; H. N. Long, Phys. Rev. D **53** (1996) 437.

[4] Y. Okamoto and M. Yasue, Phys. Lett. B **466** (1999) 267; T. Kitabayshi and M. Yasue, Phys. Rev. D **63** (2001) 095002; Phys. Rev. D **63** (2001) 095006; Nucl. Phys. B **609** (2001) 61; Phys. Rev. D **67** (2003) 015006; J. C. Montero, C. A. de S. Pires and V. Pleitez, Phys. Rev. D **65** (2002) 095001; A. A. Gusso, C. A. de S. Pires and P. S. Rodrigues da Silva, Mod. Phys. Lett. A **18** (2003) 1849; I. Aizawa *et al.*, Phys. Rev. D **70** (2004) 015011; A. G. Dias, C. A. de S. Pires and P. S. Rodriguez da Silva, Phys. Lett. B **628** (2005) 85; D. Chang and H. N. Long, Phys. Rev. D **73** (2006) 053006; P. V. Dong, H. N. Long and D. V. Soa, Phys. Rev. D **75** (2007) 073006; F. Yin, Phys. Rev. D **75** (2007) 073010.

[5] C. A. de S. Pires and O. P. Ravinez, Phys. Rev. D **58** (1998) 035008; A. Doff and F. Pisano, Mod. Phys. Lett. A **14** (1999) 1133; Phys. Rev. D **63** (2001) 097903; P. V. Dong and H. N. Long, Int. J. Mod. Phys. A **21** (2006) 6677.

[6] J. T. Liu and D. Ng, Phys. Rev. D **50** (1994) 548; J. T. Liu, Phys. Rev. D **50** (1994) 542; H. N. Long, L. P. Trung and V. T. Van, J. Exp. Theor. Phys. **92** (2001) 548, Eksp. Teor. Fiz. **119** (2001) 633; J. A. Rodriguez and M. Sher, Phys. Rev. D **70** (2004) 117702; C. Promberger, S. S. Schatt and F. Schwab, Phys. Rev. D **75** (2007) 115007.

[7] W. A. Ponce, Y. Giraldo and L.A. Sanchez, Phys. Rev. D **67** (2003) 075001.

[8] P. V. Dong, H. N. Long, D.T. Nhung and D. V. Soa, Phys. Rev. D **73** (2006) 035004.

[9] P. V. Dong, H. N. Long and D. V. Soa, Phys. Rev. D **73** (2006) 075005.

[10] P. V. Dong, Tr. T. Huong, D. T. Huong and H. N. Long, Phys. Rev. D **74** (2006) 053003.

[11] P. V. Dong *et al.*, in Ref. [4].

[12] S. Coleman and J. Mandula, Phys. Rev. **159** (1967) 1251.

[13] See for example: J. Wess and J. Bagger, *Supersymmetry and Supergravity*, 2nd edition, Princeton University Press, Princeton NJ, (1992); H. E. Haber and G. L. Kane, Phys. Rep. **117** (1985) 75.

[14] S. Martin, *A supersymmetry primer*, [arXiv:hep-ph/9709356], v. 4.

[15] V. Silveira and A. Zee, Phys. Lett. B **161** (1985) 136; D. E. Holz and A. Zee, Phys. Lett. B **517** (2000) 239; C.P. Burgess, M. Pospelov and T. ter Veldhuis, Nucl. Phys. B **619** (2002) 709; B. C. Bento, O. Bertolami, R. Rosenfeld and L. Teodoro, Phys. Rev. D **62** (2000) 041302; J. McDonald, Phys. Rev. D **50** (1994) 3637; Phys. Rev. Lett. **88** (2002) 091304.

[16] D. N. Spergel and P. J. Steinhardt, Phys. Rev. Lett. **84** (2000) 3760.

[17] J. C. Montero, V. Pleitez and M. C. Rodriguez, Phys. Rev. D **70** (2004) 075004.

[18] P. V. Dong, D. T. Huong, M. C. Rodriguez and H. N. Long, Nucl. Phys. B **772** (2007) 150.

[19] P. V. Dong, D. T. Huong, N. T. Thuy and H. N. Long, *Higgs phenomenology of supersymmetric economical 3-3-1 model*, arXiv:0707.3712 [hep-ph].

[20] P. V. Dong, D. T. Huong, M. C. Rodriguez and H. N. Long, Eur. Phys. J. C **48** (2006) 229.

[21] D. Chang and H. N. Long in Ref. [4], see also: M. B. Tully and G. C. Joshi, Phys. Rev. D **64** (2001) 011301.

[22] P. V. Dong and H. N. Long, Eur. Phys. J. C **42** (2005) 325.

[23] W.-M. Yao *et. al.* (Particle Data Group), J. Phys. G: Nucl. Part. Phys. **33** (2006) 1, p. 32.

## A. The F-term contribution

Here we present full $F$-term contributions to sfermion masses:

$$\begin{aligned}
F^{\chi'*}F_{\chi'} &= \frac{1}{4}\mu_{0a}\mu_{0b}\tilde{L}^*_{aL}\tilde{L}_{bL} + \frac{1}{4}\mu_{0a}\mu_\chi(\tilde{L}^*_{aL}\chi + \tilde{L}_{aL}\chi^*) \\
&\quad + \frac{1}{6}\mu_\chi\chi^*(\kappa_i\tilde{Q}_{1L}\tilde{u}^c_{iL} + \kappa'\tilde{Q}_{1L}\tilde{u}'^c_L) + \frac{1}{6}\mu_\chi\chi(\kappa_i\tilde{Q}^*_{1L}\tilde{u}^{c*}_{iL} + \kappa'\tilde{Q}^*_{1L}\tilde{u}'^{c*}_L), \qquad \text{(A.1)}\\
F^{\chi_\sigma *}F_{\chi^\sigma} &= \left[\frac{1}{6}\mu_\chi\chi'^{\sigma *}\left(\lambda_a\epsilon_{m\sigma n}\tilde{L}^m_{aL}\rho^n + \Pi_{\alpha i}\tilde{Q}_{\alpha L\sigma}\tilde{d}^c_{iL} + \Pi'_{\alpha\beta}\tilde{Q}_{\alpha L\sigma}\tilde{d}'^c_{\beta L}\right) + H.c.\right] \\
&\quad + \frac{1}{9}\lambda_a\lambda_b[(\tilde{L}^*_{aL}\tilde{L}_{bL})(\rho^*\rho) - (\tilde{L}^*_{aL}\rho)(\rho^*\tilde{L}_{bL})] \\
&= \left[\frac{1}{6}\mu_\chi\left(\lambda_a\epsilon\tilde{L}_{aL}\chi'^*\rho + \Pi_{\alpha i}\chi'^*.\tilde{Q}_{\alpha L}\tilde{d}^c_{iL} + \Pi'_{\alpha\beta}\chi'^*.\tilde{Q}_{\alpha L}\tilde{d}'^c_{\beta L}\right) + H.c.\right] \\
&\quad + \frac{1}{9}\lambda_a\lambda_b[(\tilde{L}^*_{aL}\tilde{L}_{bL})(\rho^*\rho) - (\tilde{L}^*_{aL}\rho)(\rho^*\tilde{L}_{bL})], \qquad \text{(A.2)}\\
F^{\rho_\sigma *}F_{\rho^\sigma} &= \left[\frac{1}{6}\mu_\rho\rho'^{\sigma *}\left(\lambda_a\epsilon_{mn\sigma}\tilde{L}^m_{aL}\chi^n + \lambda'_{ab}\epsilon_{mn\sigma}\tilde{L}^m_{aL}\tilde{L}^n_{bL}\right.\right. \\
&\quad \left.\left. + \pi_{\alpha i}\tilde{Q}_{\alpha L\sigma}\tilde{u}^c_{iL} + \pi'_\alpha\tilde{Q}_{\alpha L\sigma}\tilde{u}'^c_L\right) + H.c.\right] \\
&\quad + \frac{1}{9}\lambda_a\lambda_b[(\tilde{L}^*_{aL}\tilde{L}_{bL})(\chi^*\chi) - (\tilde{L}^*_{aL}\chi)(\chi^*\tilde{L}_{bL})] \\
&= \left[\frac{1}{6}\mu_\rho\left(\lambda_a\epsilon\tilde{L}_{aL}\chi\rho'^* + \lambda'_{ab}\epsilon\tilde{L}_{aL}\tilde{L}_{bL}\rho'^*\right.\right.
\end{aligned}$$

$$
\begin{aligned}
& + \ \pi_{\alpha i}\rho'^*.\tilde{Q}_{\alpha L}\tilde{u}^c_{iL} + \pi'_\alpha\rho'^*.\tilde{Q}_{\alpha L}\tilde{u}'^c_L\Big) + H.c.\Big] \\
& + \frac{1}{9}\lambda_a\lambda_b[(\tilde{L}^*_{aL}\tilde{L}_{bL})(\chi^*\chi) - (\tilde{L}^*_{aL}\chi)(\chi^*\tilde{L}_{bL})], \qquad \text{(A.3)}
\end{aligned}
$$

$$
F^{\rho'*}F_{\rho'} = \frac{1}{6}\mu_\rho\rho^*\left(\gamma_{ab}\tilde{L}_{aL}\tilde{l}^c_{bL} + \vartheta_i\tilde{Q}_{1L}\tilde{d}^c_{iL} + \vartheta'_\alpha\tilde{Q}_{1L}\tilde{d}'^c_{\alpha L}\right) + H.c., \qquad \text{(A.4)}
$$

$$
\begin{aligned}
F^{L^\sigma_{aL}*}F_{L^\sigma_{aL}} = & \Big[\frac{1}{6}\mu_{0a}\chi'^{\sigma*}\Big(\gamma_{ab}\rho'_\sigma\tilde{l}^c_{bL} + \lambda_a\epsilon_{\sigma mn}\chi^m\rho^n + 2\lambda'_{ab}\epsilon_{\sigma mn}\tilde{L}^m_{bL}\rho^n \\
& + \xi_{a\alpha j}\tilde{Q}_{\alpha L\sigma}\tilde{d}^c_{jL} + \xi'_{a\alpha\beta}\tilde{Q}_{\alpha L\sigma}\tilde{d}'^c_{\beta L}\Big) \\
& + \frac{1}{9}\Big(\gamma_{ab}\lambda_a\epsilon\rho'\chi^*\rho^*.\tilde{l}^c_{bL} + 2\gamma_{ab}\lambda'_{ab}\epsilon\rho'\tilde{L}^*_{bL}\rho^*.\tilde{l}^c_{bL} \\
& + 2\lambda_a\lambda'_{ab}[(\chi^*\tilde{L}_{bL})(\rho^*\rho) - (\rho^*\tilde{L}_{bL})(\chi^*\rho)] \\
& + \lambda_a\xi_{a\alpha j}\epsilon\tilde{Q}_{\alpha L}\chi^*\rho^*.\tilde{d}^c_{jL} + \lambda_a\xi'_{a\alpha\beta}\epsilon\tilde{Q}_{\alpha L}\chi^*\rho^*.\tilde{d}'^c_{\beta L}\Big) + H.c.\Big] \\
& + \frac{1}{9}\gamma_{ca}\gamma_{cb}\rho'^*.\rho'\tilde{l}^{c*}_{aL}\tilde{l}^c_{bL} \\
& + \frac{4}{9}\lambda'_{ca}\lambda'_{cb}[(\tilde{L}^*_{aL}\tilde{L}_{bL})(\rho^*\rho) - (\tilde{L}^*_{aL}\rho)(\rho^*\tilde{L}_{bL})], \qquad \text{(A.5)}
\end{aligned}
$$

$$
F^{l^c_{Lb}*}F_{l^c_{Lb}} = \frac{1}{9}\gamma_{ab}\gamma_{a'b}(\tilde{L}_{aL}\rho')(\tilde{L}_{a'L}\rho')^*, \qquad \text{(A.6)}
$$

$$
\begin{aligned}
F^{Q_{1L}*}F_{Q_{1L}} = & \frac{1}{18}[(w'^2 + u'^2)(\kappa_i\kappa_j\tilde{u}^{c*}_{iL}\tilde{u}^c_{jL} + \kappa'^2\tilde{u}'^{c*}_L\tilde{u}'^c_L + \kappa_i\kappa'\tilde{u}^{c*}_{iL}\tilde{u}'^c_L + \kappa_i\kappa'\tilde{u}^c_{iL}\tilde{u}'^{c*}_L) \\
& + v'^2(\vartheta_i\vartheta_j\tilde{d}^{c*}_{iL}\tilde{d}^c_{jL} + \vartheta'_\alpha\vartheta'_\beta\tilde{d}'^{c*}_{\alpha L}\tilde{d}'^c_{\beta L} + \vartheta_i\vartheta'_\alpha\tilde{d}^{c*}_{iL}\tilde{d}'^c_{\alpha L} + \vartheta_i\vartheta'_\alpha\tilde{d}^c_{iL}\tilde{d}'^{c*}_{\alpha L}), \qquad \text{(A.7)}
\end{aligned}
$$

$$
\begin{aligned}
F^{Q_{\alpha L}*}F_{Q_{\alpha L}} = & \frac{1}{18}[v^2(\pi_{\alpha i}\pi_{\alpha j}\tilde{u}^{c*}_{iL}\tilde{u}^c_{jL} + \pi'^2_\alpha\tilde{u}'^{c*}_L\tilde{u}'^c_L + \pi_{\alpha i}\pi'_\alpha\tilde{u}^{c*}_{iL}\tilde{u}'^c_L + \pi_{\alpha i}\pi'_\alpha\tilde{u}^c_{iL}\tilde{u}'^{c*}_L) \\
& + (w^2 + u^2)(\Pi_{\alpha i}\Pi_{\alpha j}\tilde{d}^{c*}_{iL}\tilde{d}^c_{jL} + \Pi'_{\alpha\beta}\Pi'_{\alpha\delta}\tilde{d}'^{c*}_{\beta L}\tilde{d}'^c_{\delta L} \\
& + \Pi_{\alpha i}\Pi'_{\alpha\beta}\tilde{d}^{c*}_{iL}\tilde{d}'^c_{\beta L} + \Pi_{\alpha i}\Pi'_{\alpha\beta}\tilde{d}^c_{iL}\tilde{d}'^{c*}_{\beta L})] + \cdots, \qquad \text{(A.8)}
\end{aligned}
$$

$$
\begin{aligned}
F^{u^c_{iL}*}F_{u^c_{iL}} = & \frac{1}{9}\Big[\kappa_i^2(\chi'\widetilde{Q}_{1L})(\chi'\widetilde{Q}_{1L})^* + \pi_{\alpha i}\pi_{\beta i}(\rho\widetilde{Q}_{\alpha L})(\rho\widetilde{Q}_{\beta L})^* \\
& + \kappa_i\pi_{\alpha i}(\chi'\widetilde{Q}_{1L})(\rho\widetilde{Q}_{\alpha L})^* + \kappa_i\pi_{\alpha i}(\chi'\widetilde{Q}_{1L})^*(\rho\widetilde{Q}_{\alpha L})\Big] + \cdots \\
= & \frac{1}{18}\Big[u'^2\kappa_i^2\widetilde{u}^*_{1L}\widetilde{u}_{1L} + w'^2\kappa_i^2\widetilde{u}'^*_L\widetilde{u}'_L + v^2\pi_{\alpha i}\pi_{\beta i}\widetilde{u}_{\alpha L}\widetilde{u}^*_{\beta L} \\
& + (u'w'\kappa_i^2\widetilde{u}^*_{1L}\widetilde{u'}_L - u'v\kappa_i\pi_{\alpha i}\widetilde{u}^*_{1L}\widetilde{u}_{\alpha L} - w'v\kappa_i\pi_{\alpha i}\widetilde{u}'^*_L\widetilde{u}_{\alpha L} + H.c.)\Big] + \cdots, \qquad \text{(A.9)}
\end{aligned}
$$

$$
\begin{aligned}
F^{u'^c_L*}F_{u'^c_L} = & \frac{1}{18}\Big[u'^2\kappa'^2\widetilde{u}^*_{1L}\widetilde{u}_{1L} + w'^2\kappa'^2\widetilde{u}'^*_L\widetilde{u}'_L + v^2\pi'_\alpha\pi'_\beta\widetilde{u}_{\alpha L}\widetilde{u}^*_{\beta L} \\
& + (u'w'\kappa'^2\widetilde{u}^*_{1L}\widetilde{u'}_L - u'v\kappa'\pi'_\alpha\widetilde{u}^*_{1L}\widetilde{u}_{\alpha L} - w'v\kappa'\pi'_\alpha\widetilde{u}'^*_L\widetilde{u}_{\alpha L} + H.c.)\Big] + \cdots, \qquad \text{(A.10)}
\end{aligned}
$$

$$
\begin{aligned}
F^{d^c_{iL}*}F_{d^c_{iL}} = & \frac{1}{18}\Big[v'^2\vartheta_i^2\widetilde{d}_{1L}\widetilde{d}^*_{1L} + u^2\Pi_{\alpha i}\Pi_{\beta i}\widetilde{d}_{\alpha L}\widetilde{d}^*_{\beta L} + w^2\Pi_{\alpha i}\Pi_{\beta i}\widetilde{d}'_{\alpha L}\widetilde{d}'^*_{\beta L} + \\
& + (uw\Pi_{\alpha i}\Pi_{\beta i}\widetilde{d}_{\alpha L}\widetilde{d}'^*_{\beta L} + v'u\vartheta_i\Pi_{\alpha i}\widetilde{d}_{1L}\widetilde{d}^*_{\alpha L} + v'w\vartheta_i\Pi_{\alpha i}\widetilde{d}_{1L}\widetilde{d}'^*_{\alpha L} + H.c.)\Big] + \cdots,
\end{aligned}
$$

$$
\begin{aligned}
F^{d'^c_{\delta L}*}F_{d'^c_{\delta L}} = & \frac{1}{18}\Big[v'^2\vartheta'^2_\delta\widetilde{d}_{1L}\widetilde{d}^*_{1L} + u^2\Pi'_{\alpha\delta}\Pi'_{\beta\delta}\widetilde{d}_{\alpha L}\widetilde{d}^*_{\beta L} + w^2\Pi'_{\alpha\delta}\Pi'_{\beta\delta}\widetilde{d}'_{\alpha L}\widetilde{d}'^*_{\beta L} + \\
& + (uw\Pi'_{\alpha\delta}\Pi'_{\beta\delta}\widetilde{d}_{\alpha L}\widetilde{d}'^*_{\beta L} + v'u\vartheta'_\delta\Pi'_{\alpha\delta}\widetilde{d}_{1L}\widetilde{d}^*_{\alpha L} + v'w\vartheta'_\delta\Pi'_{\alpha\delta}\widetilde{d}_{1L}\widetilde{d}'^*_{\alpha L} + H.c.)\Big] + \cdots.
\end{aligned}
$$